SECTION 6
ANALYTICAL PREDICTION OF RESPONSE

6.1 Time History Response Analysis

The time history analysis of a multi-degree-of-freedom structure subjected to earthquake excitation begins with the equations of motion for the lumped mass model which have been given previously as Equations 4-26 to 4-28. The mass matrix, $[M]$, is diagonal. The stiffness matrix, $[K]$, and damping matrix, $[C_u]$, are constructed either analytically or, as in the case of a model structure, from experimentally determined values of frequencies, damping ratios, and mode shapes (see Equations 4-24 and 4-25).

Application of Fourier Transform to Equations 4-26 to 4-28 results in

$$\{\overline{u}\} = -\,[S]^{-1}[M]\{1\}\,\overline{u}_g \tag{6-1}$$

and

$$\overline{P}_j = \frac{i\omega C_{oj}\cos^2\theta_j}{1 + i\omega\lambda}\,(\overline{u}_j - \overline{u}_{j-1}) \tag{6-2}$$

Matrix $[S]$ is given by Equations 4-30 to 4-34. Equations 6-1 and 6-2 can be solved numerically for the relative displacement vector, $\{u\}$, and horizontal component of damper force, P_j, by employing the discrete Fourier transform in combination with the Fast Fourier transform (Veletsos and Ventura 1985):

$$\{u(t)\} = \frac{-1}{2\pi}\int_{-\infty}^{\infty}[S]^{-1}[M]\{1\}\,\overline{u}_g\,e^{i\omega t}\,d\omega \tag{6-3}$$

$$P_j(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{P}_j\,e^{i\omega t}\,d\omega \tag{6-4}$$

Relative acceleration vectors are determined by an expression identical to Equation 6-3 but with the term $-\omega^2$ multiplying the inverse of the dynamic stiffness matrix, $[S]^{-1}$. The total acceleration vector, $\{\ddot{u}_t\}$, is then obtained from

$$\{\ddot{u}_t\} = \{\ddot{u}\} + \{1\}\ddot{u}_g \tag{6-5}$$

The computed total acceleration histories are used in the calculation of the story shear forces.

The time-history analysis for a single-degree-of-freedom structure is similar to the above development but with some simplifications.

6.2 Comparison of Experimental and Analytical Time History Responses

Experimental results are compared with the analytical results for the one-story unstiffened and stiffened structures in Figures 6-1 through 6-3 and Figures 6-4 through 6-5, respectively. The base shear force versus drift and the total axial damper force versus axial damper displacement are plotted for selected tests. The comparisons show good agreement.

It should be noted that, in general, the damper force - damper displacement loops (e.g., see Figures 6-2(b) and 6-4(b) are predicted very well from the analytical model. However, the analytical model tends to underpredict the story drift. The reader should recall that the two displacements were directly measured by different instruments which recorded a difference between the two quantities (see Tables 5-I and 5-II). The difference was caused by slippage in the joints of the braces. This slippage was not accounted for in the analytical model.

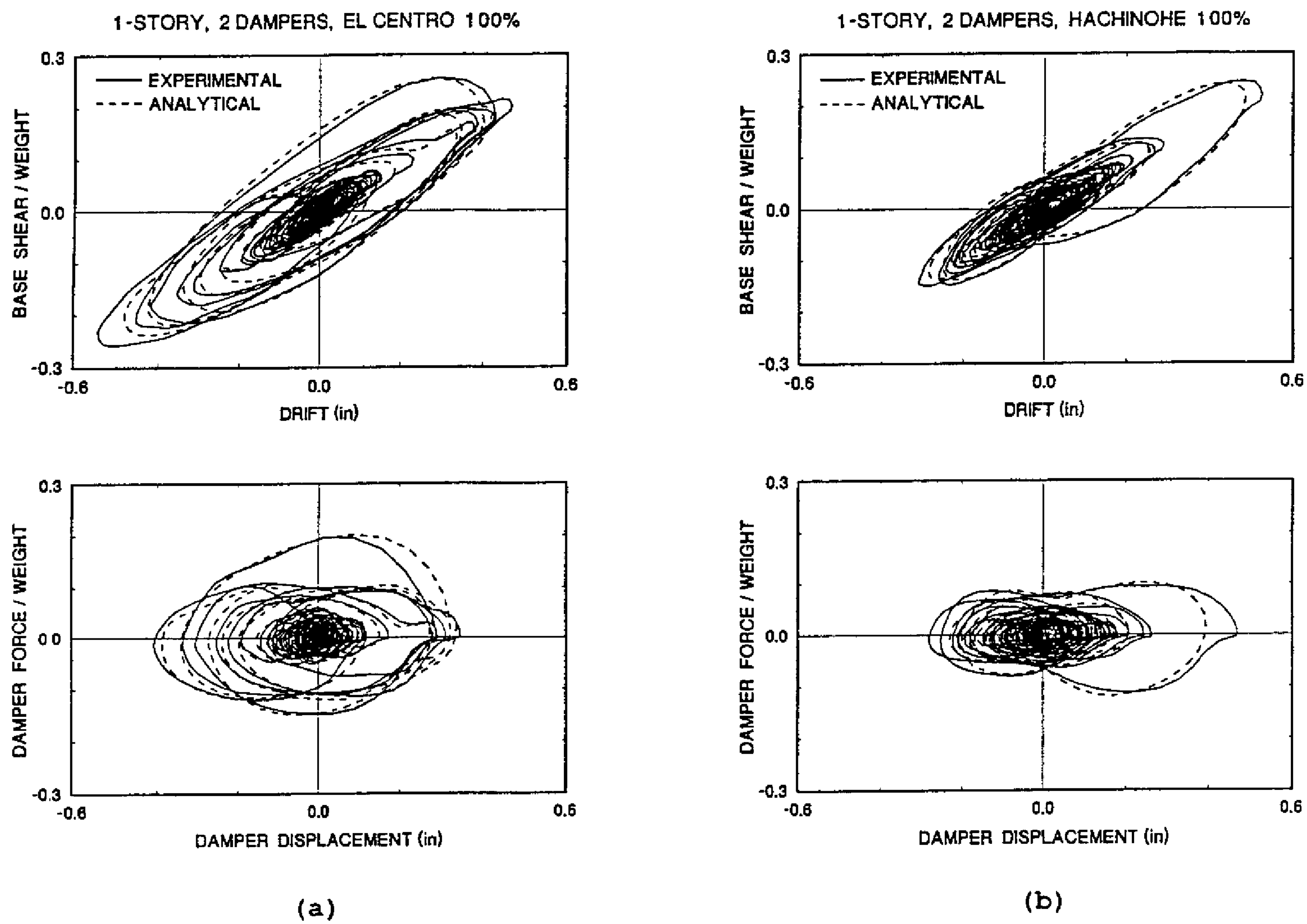


FIGURE 6-1 Comparison of Experimental and Analytical Results for the One-story Unstiffened Structure with Two Dampers Subjected to (a) El Centro 100% Motion and (b) Hachinohe 100% Motion (1 in. = 25.4 mm)

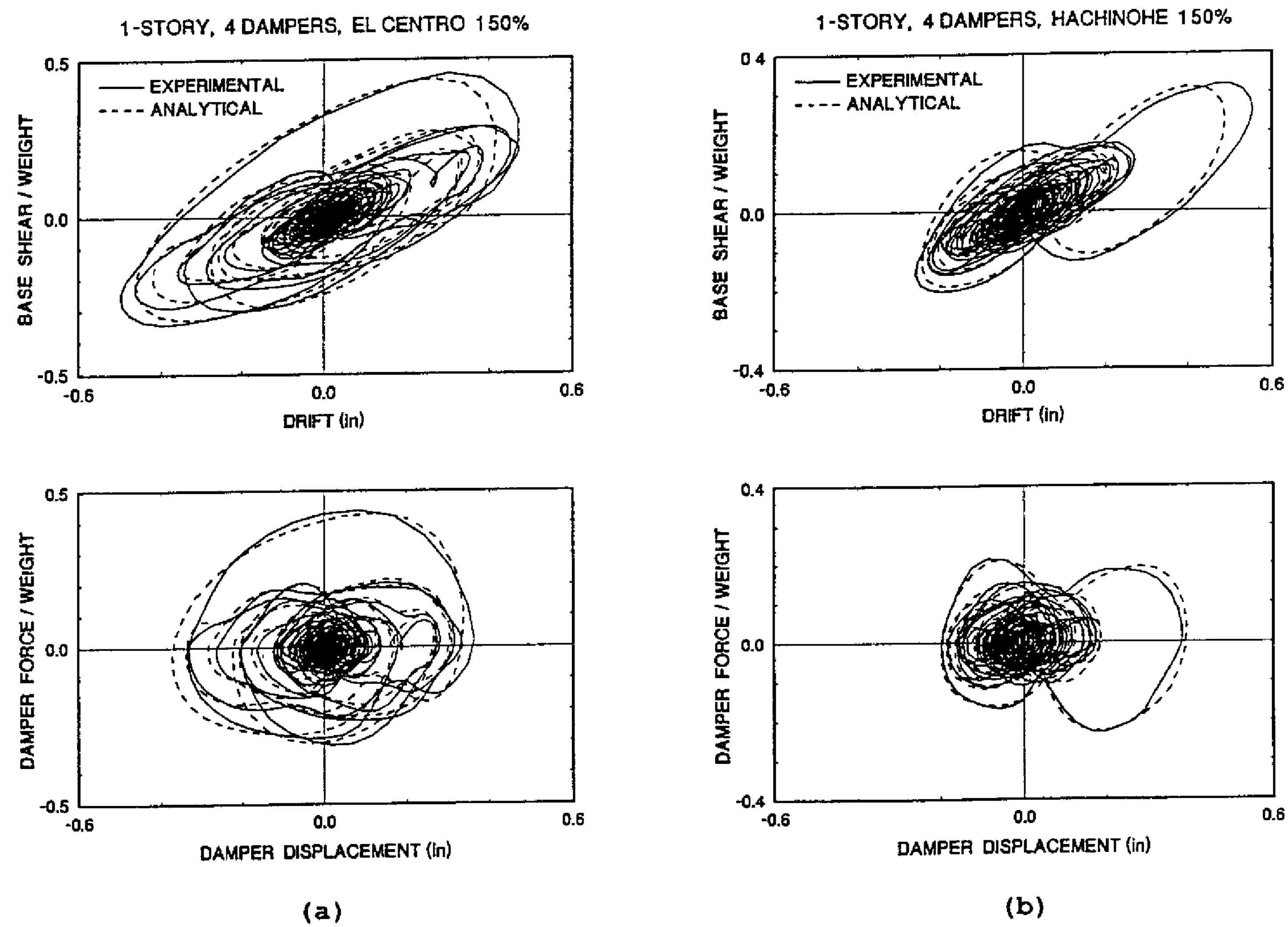


FIGURE 6-2 Comparison of Experimental and Analytical Results for the One-story Unstiffened Structure with Four Dampers Subjected to (a) El Centro 150% Motion and (b) Hachinohe 150% Motion (1 in. = 25.4 mm)

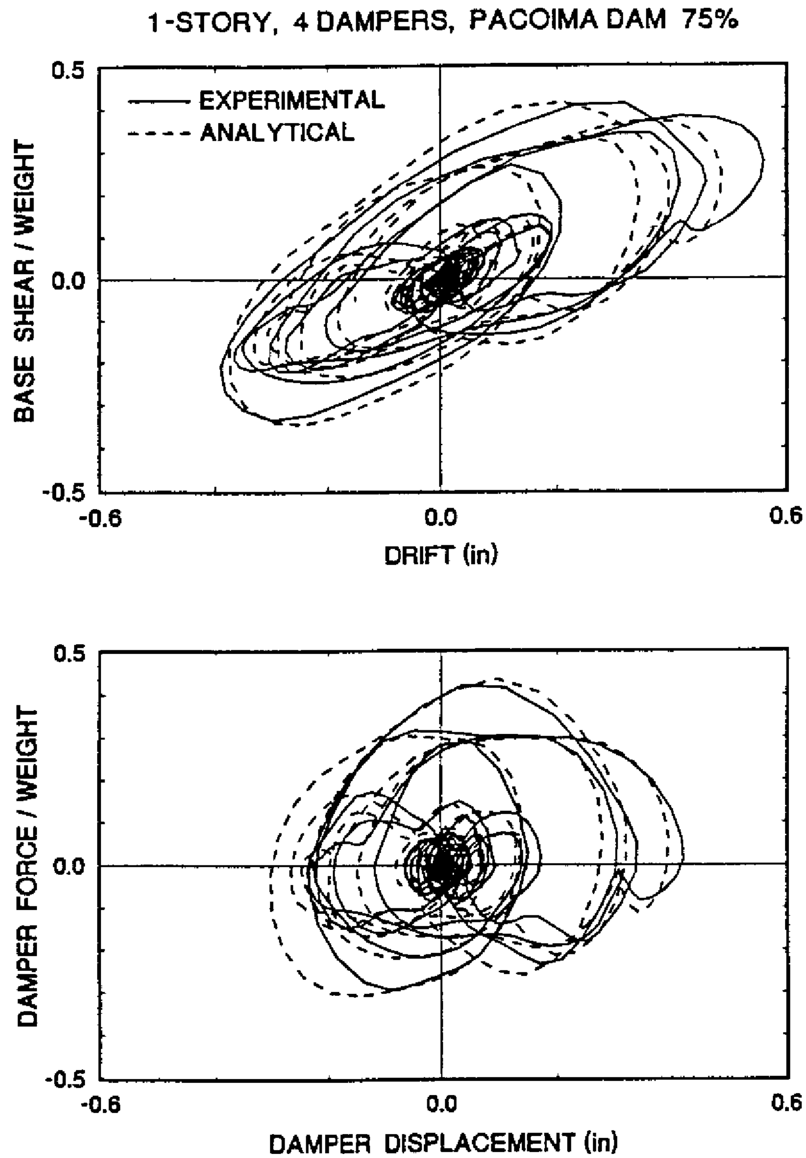
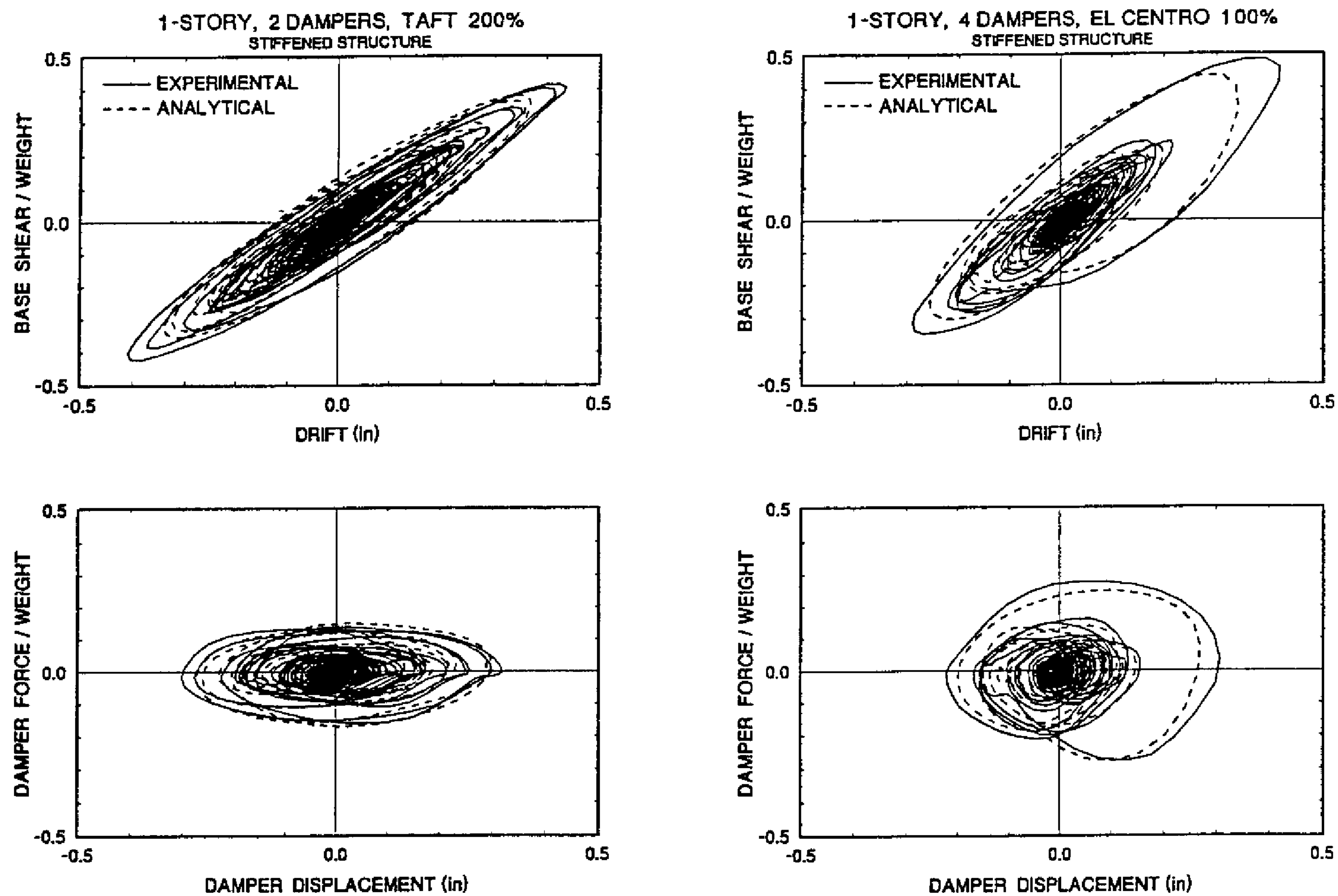


FIGURE 6-3 Comparison of Experimental and Analytical Results for the One-story Unstiffened Structure with Four Dampers Subjected to Pacoima 75% Motion (1 in. = 25.4 mm)



(a) (b)

FIGURE 6-4 Comparison of Experimental and Analytical Results for the One-story Stiffened Structure with (a) Two Dampers Subjected to Taft 200% Motion and (b) Four Dampers Subjected to El Centro 100% Motion (1 in. = 25.4 mm)

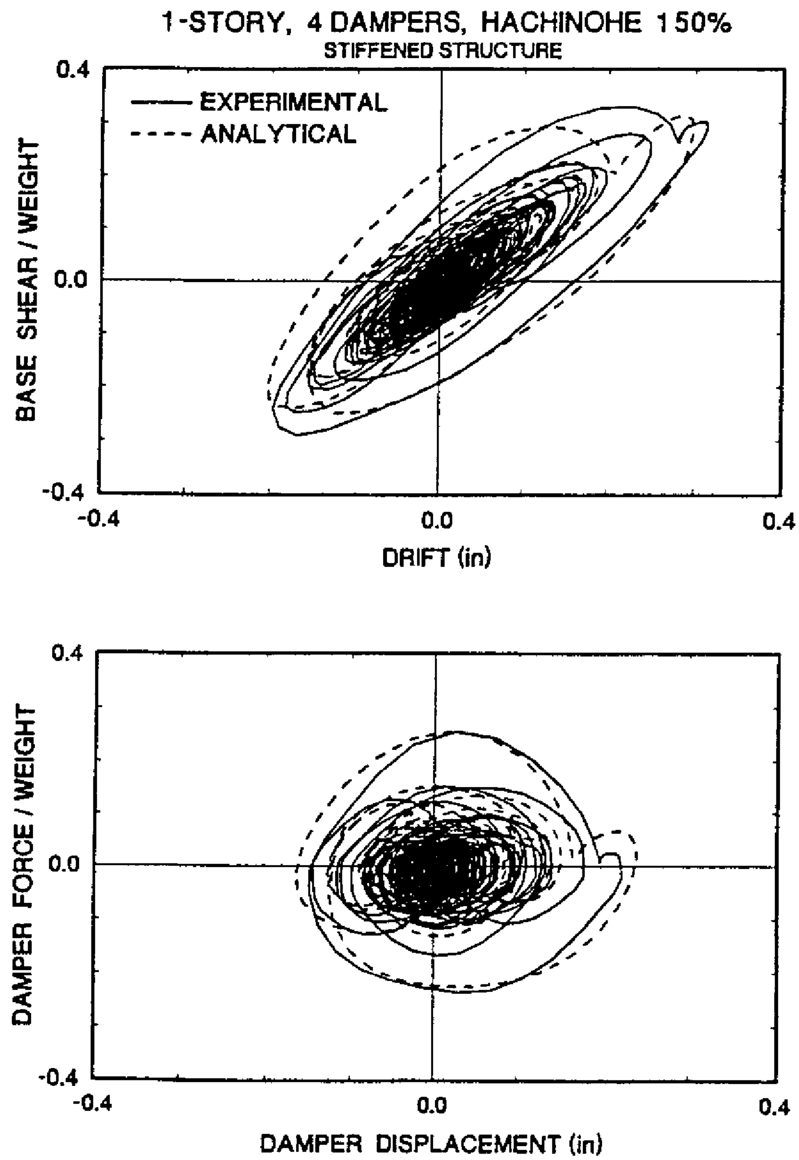


FIGURE 6-5 Comparison of Experimental and Analytical Results for the One-story Stiffened Structure with Four Dampers Subjected to Hachinohe 150% Motion (1 in. = 25.4 mm)

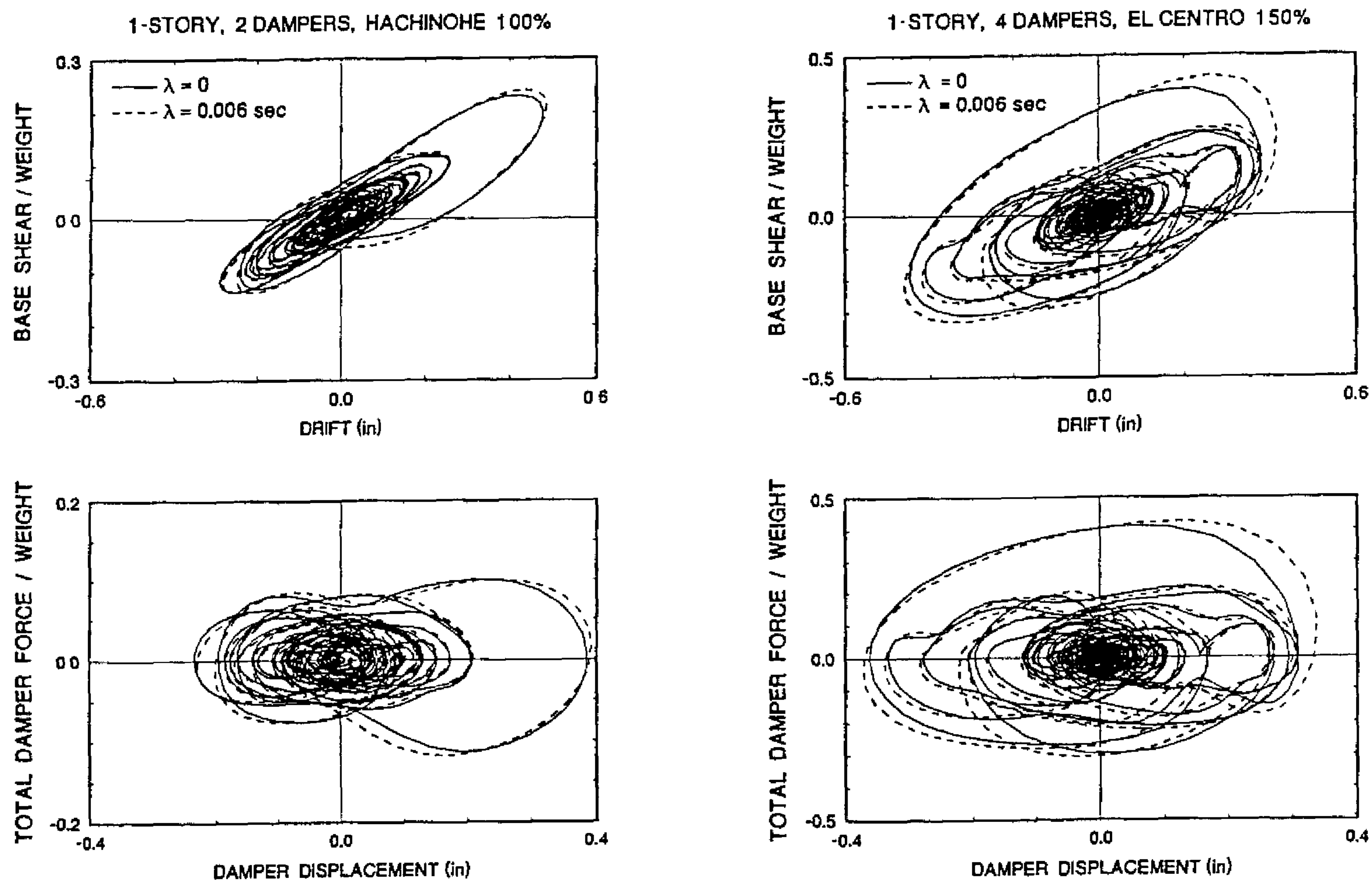
The analytical response is compared for the cases of $\lambda = 0$ (viscous model) and $\lambda = 0.006$ secs (Maxwell model) in Figures 6-6 and 6-7. The base shear force versus drift and the total axial damper force versus the axial damper displacement are plotted for selected tests. The comparisons show that approximating the damper behavior as purely viscous ($\lambda = 0$) will give nearly identical results to the more accurate viscoelastic behavior.

Comparisons of analytical and experimental story shear force versus story drift loops of the 3-story structure are presented in Figures 6-8 through 6-11. Furthermore, Figures 6-12 and 6-13 compare loops of the total axial damper force at the first story versus axial damper displacement in the 3-story structure. Again, the comparison shows good agreement.

Finally, Figures 6-14 and 6-15 compare analytical results obtained with the Maxwell model ($\lambda = 0.006$ secs) and the simple viscous model ($\lambda = 0$) representing the behavior of the fluid dampers. The first story shear versus drift and the total axial damper force at the first story versus the axial damper displacement are plotted for selected tests. These figures confirm that the simple viscous model is appropriate for analysis.

6.3 Response Spectrum Analysis Method

The comparison of analytical to experimental results in Section 6.2 demonstrated that the simple viscous model for fluid dampers is sufficiently accurate. In this respect, a structure with added fluid dampers may be modeled as a non-proportionally viscously damped system. This enables the development of an approximate method of analysis using response spectra. The advantage of this method over a time history analysis is that it directly gives the peak response by use of the usual design specification (i.e., the design spectrum).



(a) (b)

FIGURE 6-6 Comparison of Analytical Results with the Viscous (λ = 0) and Maxwell (λ = 0.006 secs) Models for the One-story Unstiffened Structure with (a) Two Dampers Subjected to Hachinohe 100% Motion and (b) Four Dampers Subjected to El Centro 150% Motion (1 in. = 25.4 mm)

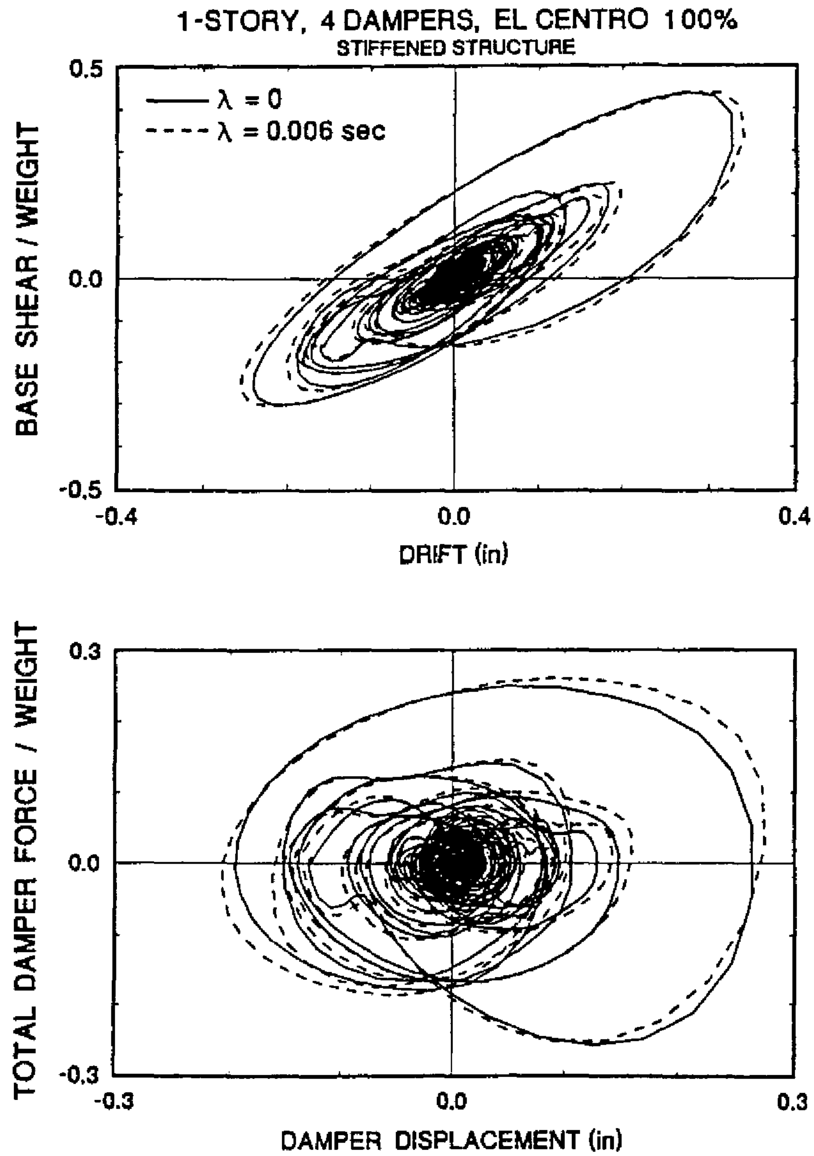


FIGURE 6-7 Comparison of Analytical Results with the Viscous (λ = 0) and Maxwell (λ = 0.006 secs) Models for the One-story Stiffened Structure with Four Dampers Subjected to El Centro 100% Motion (1 in. = 25.4 mm)

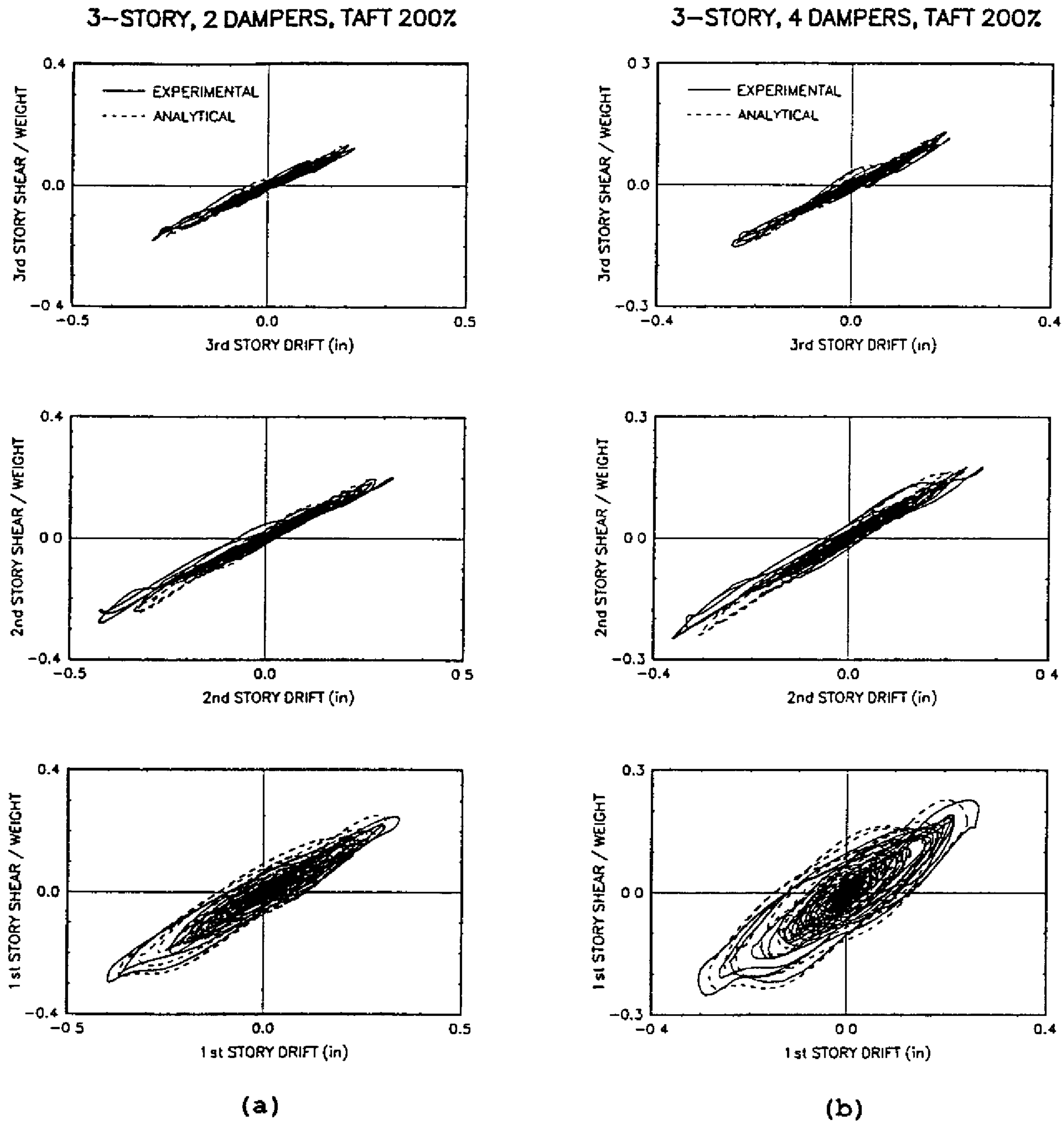


FIGURE 6-8 Comparison of Experimental and Analytical Results for the 3-story Structure Subjected to Taft 200% Motion with (a) Two Dampers and (b) Four Dampers (1 in. = 25.4 mm)

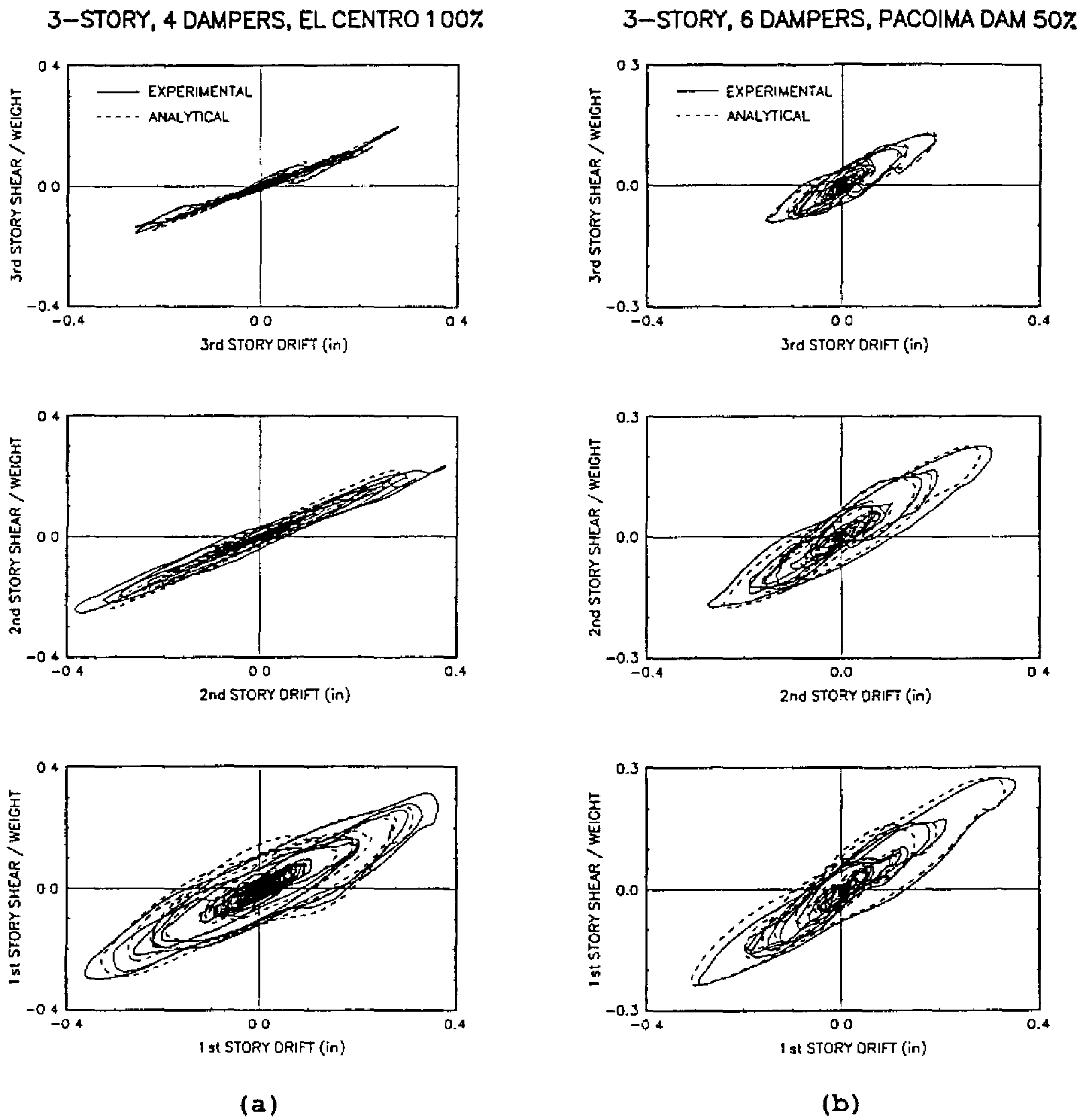


FIGURE 6-9 Comparison of Experimental and Analytical Results for the 3-story Structure with (a) Four Dampers Subjected to El Centro 100% Motion and (b) Six Dampers Subjected to Pacoima 50% Motion (1 in. = 25.4 mm)

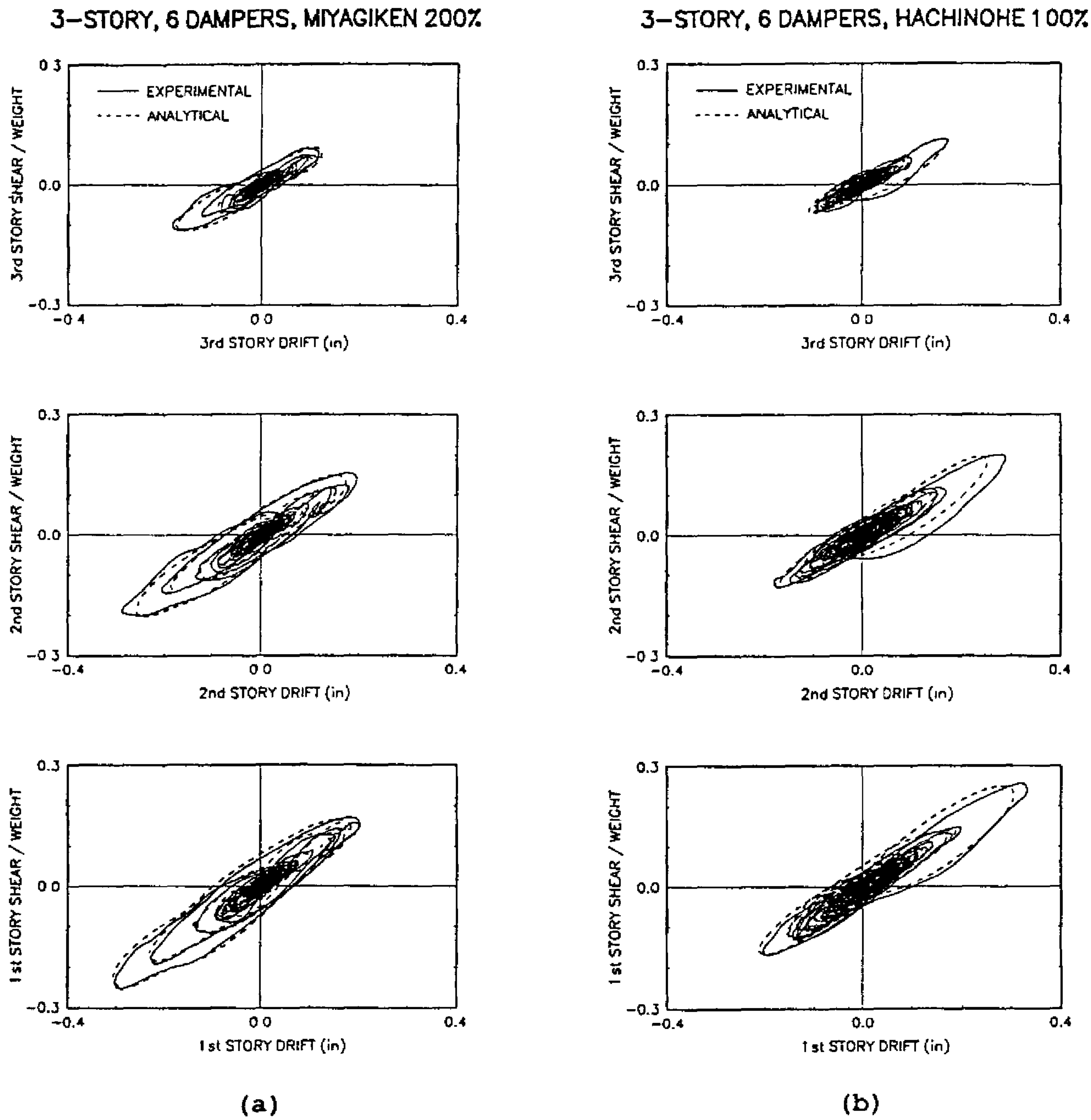


FIGURE 6-10 Comparison of Experimental and Analytical Results for the 3-story Structure with Six Dampers Subjected to (a) Miyagiken 200% Motion and (b) Hachinohe 100% Motion (1 in. = 25.4 mm)

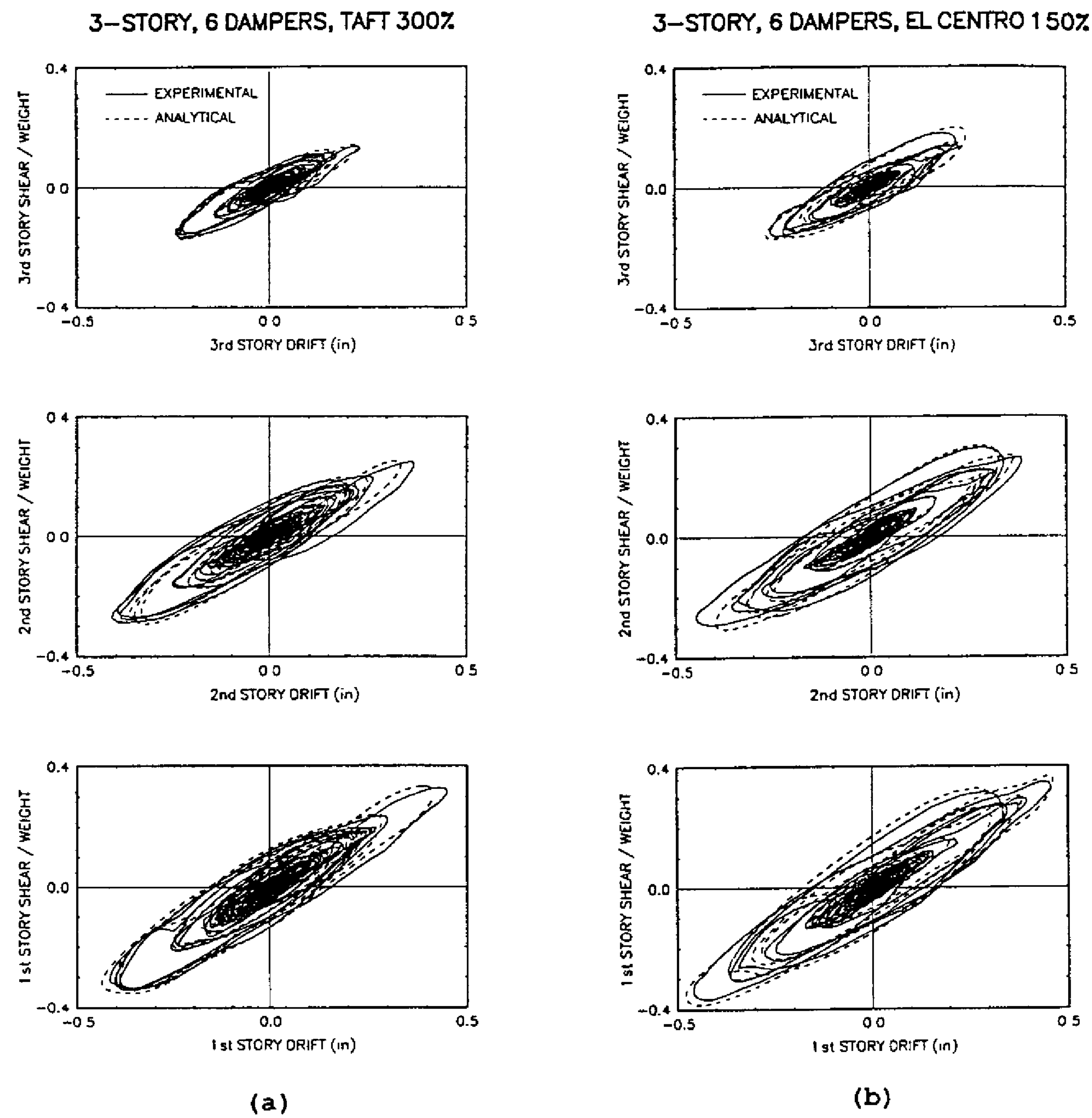


FIGURE 6-11 Comparison of Experimental and Analytical Results for the 3-story Structure with Six Dampers Subjected to (a) Taft 300% Motion and (b) El Centro 150% Motion (1 in. = 25.4 mm)

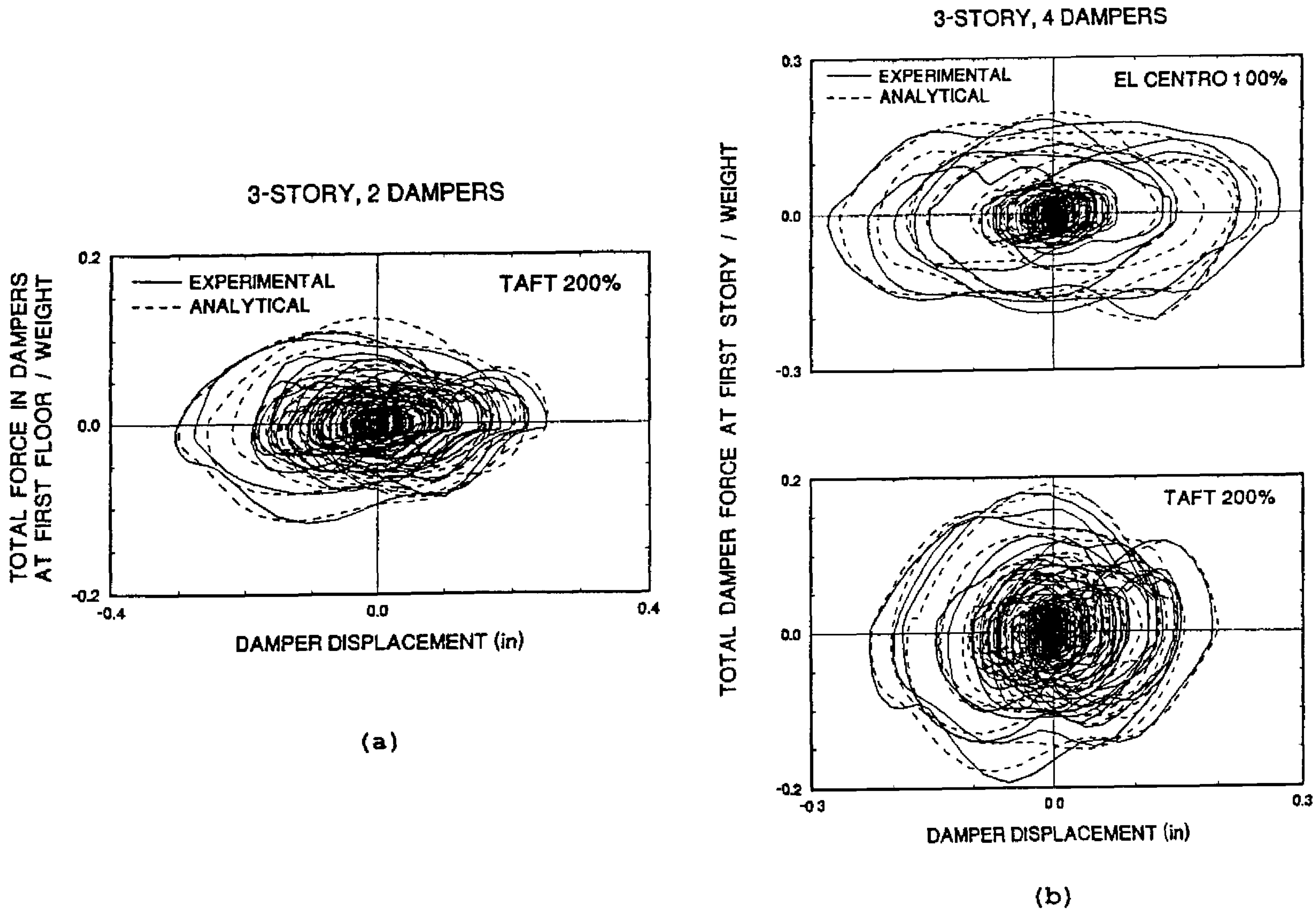


FIGURE 6-12 Comparison of Experimental and Analytical Results for the Dampers in the 3-story Structure with (a) Two Dampers and (b) Four Dampers (1 in. = 25.4 mm)

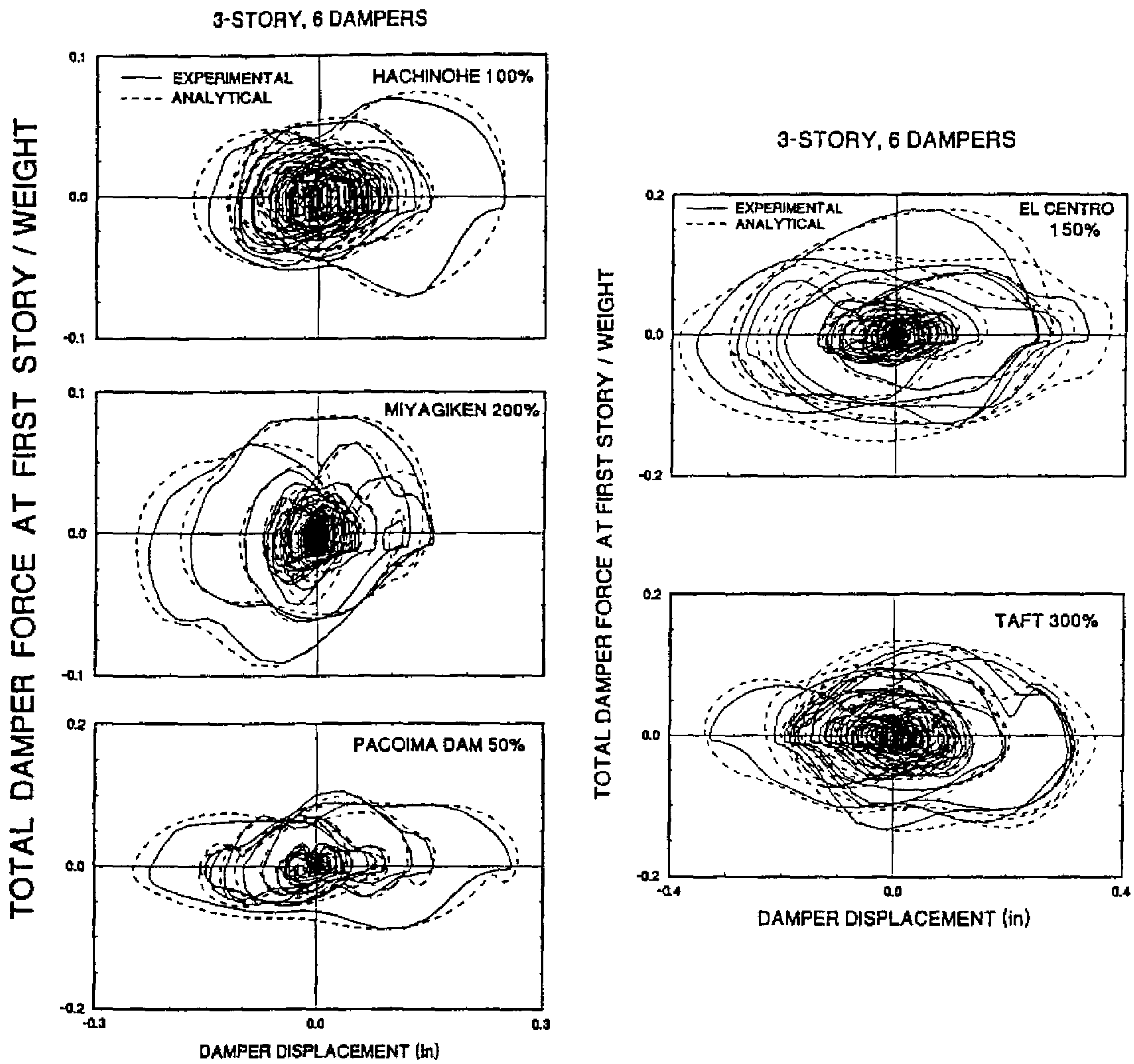


FIGURE 6-13 Comparison of Experimental and Analytical Results for the Dampers in the 3-story Structure with Six Dampers (1 in. = 25.4 mm)

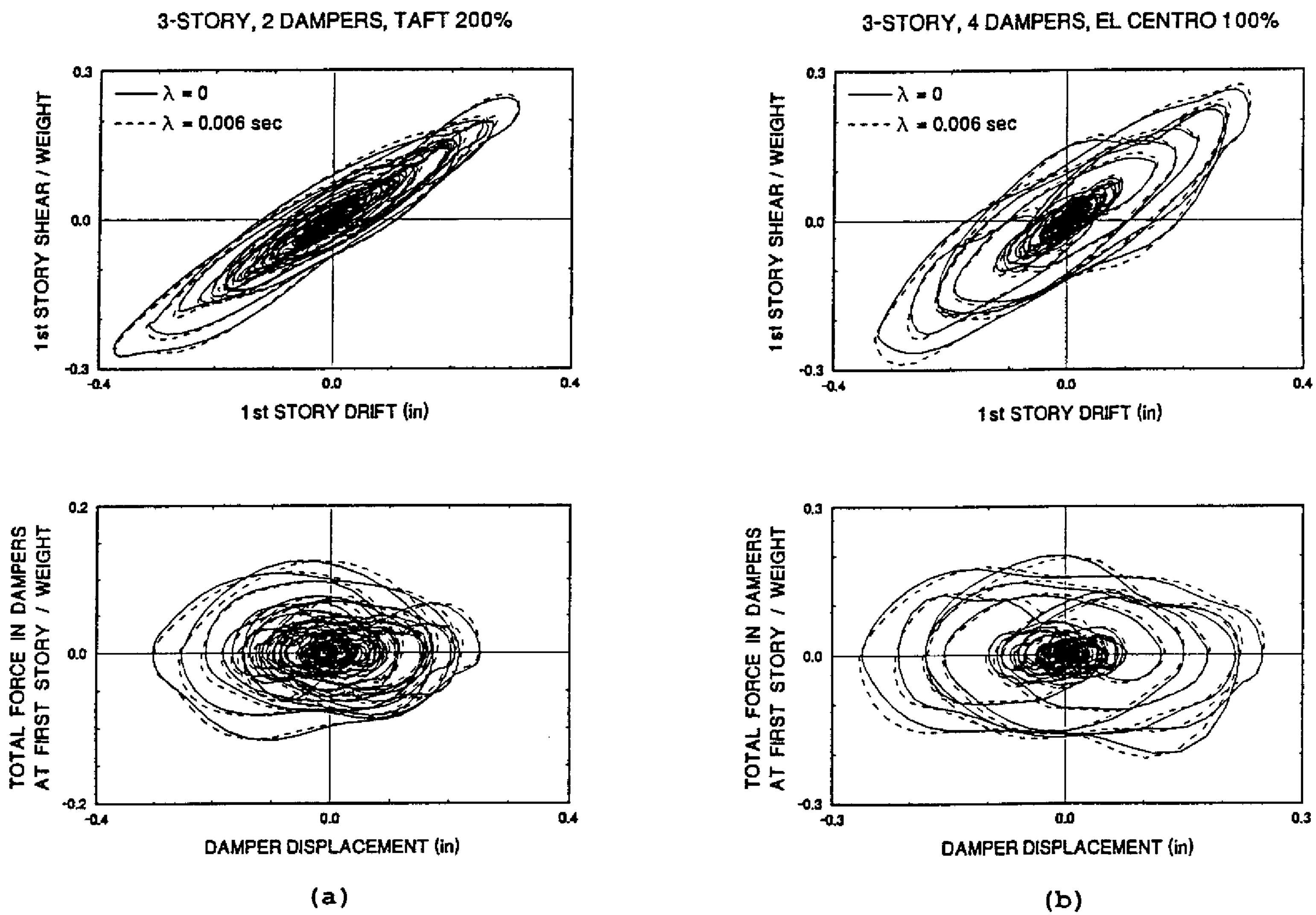


FIGURE 6-14 Comparison of Analytical Results with the Viscous (λ = 0) and Maxwell (λ = 0.006 secs) Models for the 3-story Structure with (a) Two Dampers Subjected to Taft 200% Motion and (b) Four Dampers Subjected to El Centro 100% Motion (1 in. = 25.4 mm)

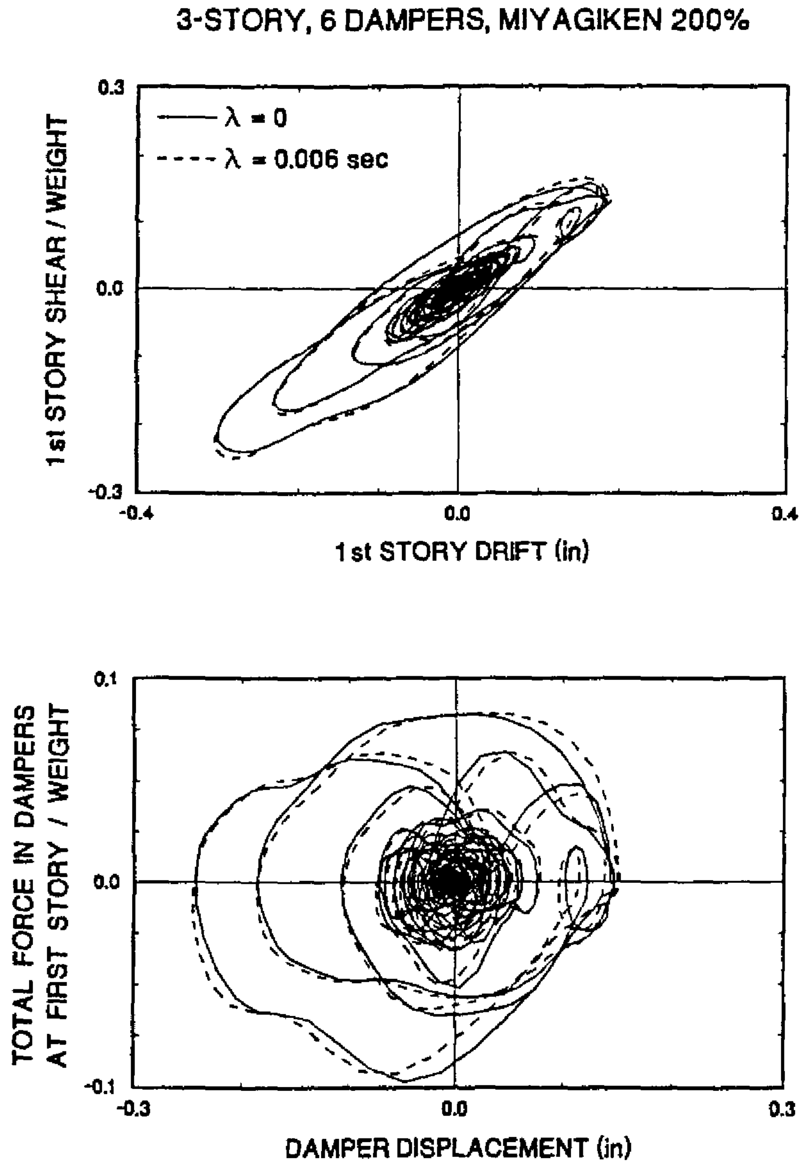


FIGURE 6-15 Comparison of Analytical Results with the Viscous ($\lambda = 0$) and Maxwell ($\lambda = 0.006$ secs) Models for the 3-story Structure with Six Dampers Subjected to Miyagiken 200% Motion (1 in. = 25.4 mm)

The application of the response spectrum analysis method requires that estimates of the structural properties are available.

6.3.1 Approximate Determination of Structural Properties

Approximate methods for the determination of the frequencies, mode shapes and damping ratios of non-classically damped structures have been successfully applied in problems involving soil-structure interaction (e.g., Novak 1983; Constantinou 1987). Veletsos (1986) presented a comprehensive treatment of the method.

The method starts with the assumption that frequencies and mode shapes of the non-classically damped structure are identical to those of the undamped structure. Typically, these quantities are determined in a standard eigenvalue analysis.

The modal damping ratios are determined from an analysis involving energy considerations. The damping ratio in the *k*-th mode of vibration may be expressed as

$$\xi_k = \xi_{str_k} + \frac{W_k}{4\pi L_k} \tag{6-6}$$

where ξ_{str_k} is the damping ratio due to damping inherent to the structure, W_k is the work done by the dampers in a single cycle of motion, and L_k is the maximum strain energy. W_k may be expressed as

$$W_k = \sum_j \int_0^{T_k} P_j d(u_j - u_{j-1}) \tag{6-7}$$

where P_j is the horizontal component of the force in the dampers at the *j*-th story, and u_j is the modal displacement of the *j*-th floor. For the case of purely viscous dampers, it can be shown that

$$P_j = C_j \cos^2\theta_j (\phi_j - \phi_{j-1}) \omega_k \cos(\omega_k t) \qquad (6\text{-}8)$$

where C_j is the combined damping coefficient of the dampers at the j-th story, θ_j is the angle of inclination of the dampers at the j-th story, ϕ_j is the modal displacement of the j-th floor in the k-th mode of vibration, and ω_k is the frequency of vibration in the k-th mode. Combining Equations 6-7 and 6-8, W_k can be written as

$$W_k = \pi \omega_k \sum_j C_j \cos^2\theta_j (\phi_j - \phi_{j-1})^2 \qquad (6\text{-}9)$$

The maximum strain energy is equal to the maximum kinetic energy, so that

$$L_k = (KE)_{MAX} = \frac{1}{2} \sum_j m_j \phi_j^2 \omega_k^2 \qquad (6\text{-}10)$$

Combining Equations 6-6, 6-9 and 6-10, the damping ratio of the structure in the k-th mode of vibration is determined to be

$$\xi_k = \xi_{str_k} + \frac{1}{2} \frac{\sum_j C_j \cos^2\theta_j (\phi_j - \phi_{j-1})^2}{\omega_k \sum_j m_j \phi_j^2} \qquad (6\text{-}11)$$

It is clear from Equation 6-11 that in order to have the greatest contribution to the modal damping ratio, the dampers should be placed at story levels where the modal interstory drift $(\phi_j - \phi_{j-1})$ is maximum.

The accuracy of the simple energy approach in determining the damping ratios of the tested structures is demonstrated in Tables 6-I and 6-II. The tables include the damping ratios calculated by the complex eigenvalue approach of Section 4 wherein the calibrated rigorous Maxwell model is utilized for the fluid dampers. The calculation was repeated by utilizing the simple viscous model and,

TABLE 6-I Comparison of Damping Ratios of One-Story Model Structure

STRUCTURE	NUMBER OF DAMPERS	RIGOROUS METHOD MAXWELL MODEL	RIGOROUS METHOD VISCOUS MODEL	ENERGY APPROACH VISCOUS MODEL
UNSTIFFENED	2	0.284	0.280	0.280
STIFFENED		0.193	0.192	0.192
UNSTIFFENED	4	0.577	0.554	0.554
STIFFENED		0.374	0.363	0.363

TABLE 6-II Comparison of Damping Ratios of 3-story Model Structure

NUMBER OF DAMPERS	RIGOROUS METHOD MAXWELL MODEL			RIGOROUS METHOD VISCOUS MODEL			ENERGY APPROACH VISCOUS MODEL		
	MODE 1	MODE 2	MODE 3	MODE 1	MODE 2	MODE 3	MODE 1	MODE 2	MODE 3
2	0.099	0.147	0.050	0.100	0.154	0.049	0.100	0.149	0.051
4	0.177	0.319	0.113	0.183	0.326	0.081	0.183	0.291	0.098
6	0.194	0.447	0.380	0.193	0.428	0.490	0.193	0.428	0.490

thus, solving exactly the eigenvalue problem of the non-classically damped structure (λ was set equal to zero). Finally, the procedure of Equation 6-11 was employed.

The results demonstrate that the damping in the fundamental mode is predicted very well by the energy approach. In addition, the energy approach provides reasonable approximations to the damping ratios of the higher modes. The error in the calculation of the higher mode damping ratios is due to neglect of the stiffening effect of the tested fluid dampers at frequencies exceeding about 4 Hz.

6.3.2 Determination of Peak Response

The determination of the peak structural response to an excitation described by a response spectrum requires that the peak response in each significant mode of vibration be evaluated first (Clough 1975). The required mode shapes, frequencies and damping ratios are determined by the procedures described in Section 6.3.1. The calculated peak modal responses are then combined by an appropriate combination rule to give estimates of the peak response.

The only complexity in the application of this approach is that of constructing high damping response spectra from the usually specified 5%-damped spectra. A recent study on this problem has been reported by Wu (1989). However, it may be appropriate to include de-amplification factors of design spectra at high damping in future design requirements of structures with supplemental damping devices. This will ensure uniformity, reasonable conservatism and avoidance of gross errors.

6.4 Comparison of Experimental, Time History, and Response Spectrum Results

Comparisons of peak response of interest in design (i.e., story shear forces and interstory drifts) are presented in Tables 6-III through 6-VI for the 3-story structure with 4 dampers subjected to the Taft 200% excitation, and for the structure with 6 dampers subjected to the Miyagiken 200%, Hachinohe 100%, and El Centro 150% excitations, respectively. The peak response is given experimentally and analytically as calculated by time history analysis and by the response spectrum approach. For the application of the response spectrum approach, the high damping displacement and acceleration spectra of Figures 3-10 to 3-14 were utilized. Interpolation was used for values of damping ratio not included in these figures.

The peak responses as determined by all four methods compare well. The prediction of story shear forces is very good but the prediction of interstory drifts is less accurate. The reader should recall that slippage occurred in the joints of the damped frame. This effect was not accounted for in the analytical models. The simple response spectrum approach yields results which are accurate enough for design purposes.

TABLE 6-III Comparison of Peak Response to Taft 200% Excitation of 3-story Structure with Four Dampers as Determined Experimentally and by Various Analytical Methods

RESPONSE	EXPERIMENTAL	TIME HISTORY, MAXWELL MODEL	TIME HISTORY, VISCOUS MODEL	RESPONSE SPECTRUM APPROACH
3rd Story Shear / *Weight*	0.155	0.142	0.137	0.113
2nd Story Shear / *Weight*	0.249	0.240	0.227	0.186
1st Story Shear / *Weight*	0.253	0.237	0.231	0.231
3rd Story Drift / *Height* (%)	0.829	0.753	0.726	0.571
2nd Story Drift / *Height* (%)	1.208	1.026	0.977	0.829
1st Story Drift / *Height* (%)	0.949 (0.887)*	0.882	0.884	0.933

* = Measured from relative displacement of damper

TABLE 6-IV Comparison of Peak Response to Miyagiken 200% Excitation of 3-story Structure with Six Dampers as Determined Experimentally and by Various Analytical Methods

RESPONSE	EXPERIMENTAL	TIME HISTORY, MAXWELL MODEL	TIME HISTORY, VISCOUS MODEL	RESPONSE SPECTRUM APPROACH
$\frac{\textit{3rd Story Shear}}{\textit{Weight}}$	0.114	0.116	0.114	0.106
$\frac{\textit{2nd Story Shear}}{\textit{Weight}}$	0.202	0.204	0.197	0.178
$\frac{\textit{1st Story Shear}}{\textit{Weight}}$	0.254	0.250	0.241	0.221
$\frac{\textit{3rd Story Drift}}{\textit{Height}}$ (%)	0.610	0.583	0.586	0.540
$\frac{\textit{2nd Story Drift}}{\textit{Height}}$ (%)	0.963	0.859	0.858	0.788
$\frac{\textit{1st Story Drift}}{\textit{Height}}$ (%)	0.947 (0.890)*	0.952	0.942	0.885

* = Measured from relative displacement of damper

TABLE 6-V Comparison of Peak Response to Hachinohe 100% Excitation of 3-story Structure with Six Dampers as Determined Experimentally and by Various Analytical Methods

RESPONSE	EXPERIMENTAL	TIME HISTORY, MAXWELL MODEL	TIME HISTORY, VISCOUS MODEL	RESPONSE SPECTRUM APPROACH
3rd Story Shear / Weight	0.111	0.112	0.106	0.108
2nd Story Shear / Weight	0.201	0.195	0.186	0.185
1st Story Shear / Weight	0.256	0.250	0.243	0.229
3rd Story Drift / Height (%)	0.575	0.566	0.558	0.563
2nd Story Drift / Height (%)	0.963	0.837	0.825	0.828
1st Story Drift / Height (%)	1.036 (0.957) *	0.954	0.946	0.927

* = Measured from relative displacement of damper

TABLE 6-VI Comparison of Peak Response to El Centro 150% Excitation of 3-story Structure with Six Dampers as Determined Experimentally and by Various Analytical Methods

RESPONSE	EXPERIMENTAL	TIME HISTORY, MAXWELL MODEL	TIME HISTORY, VISCOUS MODEL	RESPONSE SPECTRUM APPROACH
3rd Story Shear / *Weight*	0.178	0.198	0.177	0.176
2nd Story Shear / *Weight*	0.298	0.307	0.295	0.294
1st Story Shear / *Weight*	0.368	0.387	0.368	0.365
3rd Story Drift / *Height* (%)	0.852	0.889	0.876	0.896
2nd Story Drift / *Height* (%)	1.492	1.323	1.302	1.306
1st Story Drift / *Height* (%)	1.436 (1.382)*	1.498	1.465	1.468

* = Measured from relative displacement of damper

SECTION 7
CONCLUSIONS

A combined experimental and analytical study of an energy absorbing system for structures, consisting of fluid viscous dampers, has been presented. Tests were conducted on one- and 3-story model structures with various configurations of dampers. Dampers were placed either along the entire height of the 3-story structure, or concentrated at the level of expected peak interstory drift. Tests were also conducted on the bare frame in a configuration resembling a moment resisting frame.

A comprehensive component test program on the fluid dampers was conducted. The test program evaluated the behavior of the dampers in a range of frequencies varying between essentially zero and 25 Hz, a range of amplitudes of essentially zero to 1 inch (25.4 mm), and a range of temperatures between about zero and 50°C. The component tests resulted in a database of mechanical properties which enabled the development of a rigorous mathematical model.

The mathematical model was utilized in the time history analysis of the tested structures with very good results. Furthermore, simplified models and methods of analysis were developed, evaluated and shown to produce results in good agreement with the experiments.

The important conclusions of this study are summarized below:

a) Fluid viscous dampers may be designed to exhibit a behavior which is essentially linear viscous for frequencies of motion below a certain cutoff frequency. For the tested damper this frequency was equal to about 4 Hz. Beyond this frequency the dampers exhibit viscoelastic behavior.

b) Fluid dampers may be modeled over a wide range of frequencies by the classical Maxwell model. However, since the cutoff frequency is usually above (or they can be designed so) the frequencies of dominant modes of the structure, the dampers may be modeled as simple linear viscous dampers.

c) Temperature has a minor effect on the behavior of the tested fluid dampers. Due to a special design with a passive temperature-compensated orifice, the tested dampers exhibited variations of their damping constant from a certain value at room temperature (24°C) to +44% of that value at 0°C to -25% of that value at 50°C. This rather small change in properties over a wide range of temperatures is in sharp contrast to the extreme temperature sensitivity of viscoelastic dampers.

d) The inclusion of fluid viscous dampers in the tested structures resulted in reductions in story drifts of 30% to 70%. These reductions are comparable to those achieved by other energy dissipating systems such as viscoelastic, friction and yielding steel dampers. However, the use of fluid dampers also resulted in reductions of story shear forces by 40% to 70%, while other energy absorbing devices were incapable of achieving any significant reduction.

e) Fluid dampers are capable of achieving and surpassing the benefits offered by active control systems with the additional benefits of low cost, no requirements for power, longevity and reliability.

f) Due to their viscous nature, fluid dampers reduce drifts and thus column bending moments, while introducing additional column axial forces which are out-of-phase with the bending moments. In effect, this behavior prevents the possibility of compression failure of weak columns in retrofit applications.

g) Time history analyses of structures with added fluid dampers may be more conveniently performed by application of Discrete Fourier transform since the dampers exhibit linear behavior. Such analyses were performed for the tested structure with the results being in good agreement with the results of the experiments.

h) A simplified method for calculating the modal characteristics of structures with added fluid dampers was developed and verified. The method was used to obtain estimates of peak response of the tested structures by utilizing the response spectrum approach. The obtained results demonstrated that the simplified method is sufficiently accurate for design purposes.

SECTION 8
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