A GENERALIZATION OF OPTIMAL CONTROL THEORY: NONLINEAR CONTROL

PART II

SECTION 1
INTRODUCTION

A combined use of active and passive control systems, referred to as the hybrid control system, has been demonstrated to be very effective for seismic-excited civil engineering structures. Various hybrid control systems have been investigated for applications to protect building and bridge structures [e.g., 2-5, 7-8, 13-22 and Refs. in Part I]. However, the application of hybrid control systems involves active control of nonlinear or hysteretic structures, since most passive control devices, such as lead-core rubber-bearing isolation systems, behave either nonlinearly or inelastically.

Control laws can either be linear or nonlinear. Linear control theories for linear structures have been available in the literature. However, control theories for nonlinear structures are limited and intensive research efforts have been made. Recently, instantaneous optimal control has been proposed for applications to nonlinear and hysteretic structures successfully [e.g., 18-21]. In particular, the stable controllers are obtained by use of the Lyapunov direct method [e.g., 20-21]. Basically, the control law proposed in these previous works is the linear control law. Various control laws for discrete pulse control, that is nonlinear in nature, have been proposed for applications to nonlinear civil engineering structures [e.g., 6, 7].

Another control method proposed for applications to buildings equipped with frictional-type sliding isolation systems is the method of dynamic linearization [Ref. 22]. The method of dynamic linearization is to synthesize the control vector so that the response of the controlled structure matches that of a specified system, referred to as the template system [e.g., 1, 9], whereas the response characteristics of the template system is known. This control method has been applied successfully to seismic-excited buildings equipped with a frictional-type sliding isolation system [Ref. 22]. However, for applications to other types of nonlinear structures, the major difficulty is to find a suitable template system such that the response of the nonlinear structure can be matched easily to that of the template system. Because of such a difficulty, the application of the dynamic linearization method is limited to a certain class of base-isolated buildings. The control law associated with the method of dynamic linearization is usually nonlinear [Ref. 22]. Other control methods have also been proposed for structures equipped with frictional-type sliding isolation systems [e.g., 2, 5, 8].

The polynomial control law has been suggested in Refs. 11-12 for applications to nonlinear structures. It was shown that the control performance of the nonlinear polynomial control law is better than that of the linear control law. However, the main disadvantage of the polynomial control law is that the computations of the gain matrices for higher order control terms are rather cumbersome.

In this report, an optimal nonlinear control law is proposed for applications to nonlinear and hysteretic structures. The proposed nonlinear control method is based on a generalized performance index. The resulting optimal control law resembles the nonlinear characteristics of the structure to be controlled. The absolute acceleration vector of the structural response is included in the generalized performance index, and the actuator dynamics is also taken into account in the optimization process. Likewise, control laws using acceleration and velocity feedbacks are derived in Section 7.

An extensive simulation study has been conducted. Simulation results indicate that (i) the proposed nonlinear control method is effective for hybrid control of some types of seismic-excited building structures, and (ii) the performance of the optimal nonlinear control method is better than that of the linear control method proposed in Refs. 18-21.

SECTION 2

OPTIMAL LINEAR CONTROL FOR NONLINEAR STRUCTURES

An instantaneous optimal control method was proposed for nonlinear and hysteretic structural systems [e.g., 18-21]. In the previous works, a discretization of the equations of motion was made to obtain an approximate solution leading to a linear control law. Then, the stability of the controllers is guaranteed by use of the Lyapunov direct method [e.g., 20-21]. Using the Lyapunov direct method, a Riccati-type equation and a Lyapunov equation were obtained for the determination of stable controllers. In this section, we shall derive the same linear control law using the LQR performance index. The nonlinear control law will be proposed in the next section.

Consider an n degrees of freedom nonlinear or hysteretic building structure subjected to a one-dimensional earthquake ground acceleration $\ddot{X}_0(t)$. The vector equation of motion is given by

$$\underline{M}\ddot{\underline{X}}(t) + \underline{F}_D[\dot{\underline{X}}(t)] + \underline{F}_s[\underline{X}(t)] = \underline{\xi}\ddot{X}_0(t) + \underline{H}_1\underline{U}(t) \qquad (2.1)$$

in which $\underline{X}(t)=[x_1,x_2,\ldots,x_n]'$ = an n-vector with $x_j(t)$ being the deformation of the jth story unit, $\underline{U}(t)$ = a r-dimensional vector consisting of r control forces, $\underline{\xi}=-[m_1,m_2,\ldots,m_n]'$ = a mass vector. In Eq. (2.1), $\underline{M}$ is a (nxn) mass matrix with the i-jth element $M(i,j)=m_i$ for $j\leq i$ and $M(i,j)=0$ for $j>i$, where m_i is the mass of the ith floor. $\underline{F}_D[\dot{\underline{X}}(t)]$ and $\underline{F}_s[\underline{X}(t)]$ are nonlinear damping and stiffness vectors, respectively, and $\underline{H}_1$ is a (nxr) matrix denoting the location of r controllers. In the notation above, an under bar denotes either a vector or a matrix and a prime indicates the transpose of either a matrix or a vector.

In the state vector form, Eq. (2.1) becomes

$$\dot{\underline{Z}}(t) = \underline{g}[\underline{Z}(t)] + \underline{B}\underline{U}(t) + \underline{W}_1\ddot{X}_0(t) \qquad (2.2)$$

in which $\underline{g}[\underline{Z}(t)]$ is a 2n vector which is a nonlinear function of the state vector $\underline{Z}(t)$ and

$$\underline{Z}(t) = \begin{bmatrix} \underline{X}(t) \\ --- \\ \dot{\underline{X}}(t) \end{bmatrix};\ \underline{B} = \begin{bmatrix} \underline{0} \\ --- \\ \underline{M}^{-1}\underline{H}_1 \end{bmatrix};\ \underline{W}_1 = \begin{bmatrix} \underline{0} \\ --- \\ \underline{M}^{-1}\underline{\xi} \end{bmatrix};\ \underline{g}[\underline{Z}(t)] = \begin{bmatrix} \dot{\underline{X}}(t) \\ -------- \\ -\underline{M}^{-1}[\underline{F}_D + \underline{F}_s] \end{bmatrix} \qquad (2.3)$$

The LQR performance index is given by

$$J = \int_0^{t_f} \left[Z'(t) Q Z(t) + \underline{U}'(t) \underline{R} \underline{U}(t) \right] dt \tag{2.4}$$

in which $\underline{Q}$ is a (2nx2n) symmetric positive semidefinite weighting matrix and $\underline{R}$ is a (rxr) positive definite weighting matrix.

To minimizing the objective function, J, given by Eq. (2.4) subjected to the constraint of the state equation of motion, Eq. (2.2), the Hamiltonian H is constructed by introducing a 2n-dimensional Lagrangian multiplier vector $\underline{\lambda}$(t),

$$H = \underline{Z}' Q \underline{Z} + \underline{U}' \underline{R} \underline{U} + \underline{\lambda}' [\underline{g}(\underline{Z}) + \underline{B} \underline{U} + \underline{W}_1 \ddot{X}_0 - \dot{\underline{Z}}] \tag{2.5}$$

in which the argument t has been dropped for simplicity.

The necessary conditions for minimizing J given by Eq. (2.4) are

$$\frac{\partial H}{\partial \underline{\lambda}} = 0 \quad ; \quad \frac{\partial H}{\partial \underline{U}} = 0 \quad ; \quad \frac{\partial H}{\partial \underline{Z}} + \dot{\underline{\lambda}}' = 0 \, , \tag{2.6}$$

The first condition $\partial H / \partial \underline{\lambda} = 0$ leads to the state equation of motion given by Eq. (2.2). The second and third conditions are obtained as follows

$$\underline{U} = -0.5 \underline{R}^{-1} \underline{B}' \underline{\lambda} \tag{2.7}$$

and

$$2Q\underline{Z} + \Delta'(\underline{Z}) \underline{\lambda} + \dot{\underline{\lambda}} = 0 \tag{2.8}$$

in which

$$\Delta(\underline{Z}) = \partial \underline{g}(Z) / \partial \underline{Z} \tag{2.9}$$

is a (2nx2n) derivative matrix.

Let

$$\underline{\lambda} = \underline{P} \underline{Z} \tag{2.10}$$

in which $\underline{P}$ is a (2nx2n) matrix to be determined. Substitution of Eq. (2.10) into Eqs. (2.7) and (2.8) yields

$$\underline{U}(t) = -0.5 \underline{R}^{-1} \underline{B}' \underline{P} \underline{Z} \tag{2.11}$$

$$2Q\underline{Z} + \Delta'(\underline{Z}) \underline{P} \underline{Z} + \dot{\underline{P}} \underline{Z} + \underline{P} \dot{\underline{Z}} = 0 \tag{2.12}$$

Substituting $\dot{\underline{Z}}$ given by Eq. (2.2) into Eq. (2.12), using Eq. (2.11) and neglecting the earthquake ground acceleration $X_0(t)$, one obtains

$$PZ + \Delta'(Z)PZ + Pg(Z) - 0.5PBR^{-1}B'PZ = -2QZ \qquad (2.13)$$

Equation (2.13) should be solved backwards from the terminal point t_f, i.e., $P(t_f)=0$. However, since the earthquake ground acceleration $\ddot{X}_0(t)$ is not known, i.e., $Z(t)$ is not known, Eq. (2.13) can not be solved. Consequently, an approximation using the equivalent linearization technique is used.

One possible approach is to linearize the structural system at the initial equilibrium point $Z=0$ that is stable for civil engineering structures. Hence $g(Z)$ and $\Delta(Z)$ are approximated by

$$g(Z) \simeq \Delta_0 Z$$
$$\Delta(Z) \simeq \Delta_0$$

and Eq. (2.13) becomes

$$\dot{P} + \Delta_0' P + P\Delta_0 - 0.5PBR^{-1}B'P = -2Q \qquad (2.14)$$

in which P is the Riccati matrix where

$$\Delta_0 = \Delta(Z)\big|_{Z=0} \qquad (2.15)$$

In earthquake engineering applications, it has been shown [e.g., 15, 16] that the time dependent Riccati matrix establishes its stationary values rapidly such that $\dot{P}=0$ is an excellent approximation. As a result, Eq. (2.15) can be approximated by the matrix algebraic equation

$$\Delta_0' P + P\Delta_0 - 0.5PBR^{-1}B'P = -2Q \qquad (2.16)$$

Equation (2.16) was also derived based on the Lyapunov direct method for instantaneous optimal control in Refs. 20-21. Furthermore, since the term $P\,B\,R^{-1}\,B'P$ is positive semidefinite, Eq. (2.16) can also be approximated by

$$\Delta_0' P + P\Delta_0 = -2Q \qquad (2.17)$$

which is known as the Lyapunov equation. From Eq. (2.17), various approximate solutions have been proposed in Ref. 20 for control of linear, nonlinear and hysteretic structures.

SECTION 3

OPTIMAL NONLINEAR CONTROL FOR NONLINEAR STRUCTURES

In the previous section, the LQR performance index is used and a linear control law is derived for nonlinear structures where the derivative matrix $\underline{\Lambda}(\underline{Z})$ is evaluated at the initial equilibrium point $\underline{Z}=\underline{0}$. The same solution was obtained in Ref. 20 using the Lyapunov direct method. Such an approach works well when yielding of inelastic structures is not quite serious. As the ductility becomes large, the control performance of the linear control law presented in the previous section will be examined later. In this section, two nonlinear control laws are proposed for control of nonlinear structures.

A performance index is proposed as follows

$$J = \int_0^{t_f} \left[\underline{g}'(\underline{Z}) \underline{Q} \underline{g}(\underline{Z}) + \underline{U}'(t) \underline{R} \underline{U}(t) \right] dt \tag{3.1}$$

The performance index J, proposed in Eq. (3.1), is quite different from that of LQR, Eq. (2.4), since $g(\underline{Z})$ is a nonlinear function of $\underline{Z}$, see Eq. (2.3), which is the nonlinear characteristic of the hysteretic system.

To minimize the performance index J subjected to the constraint of the matrix equation of motion, the Hamiltonian H is expressed as

$$H = \underline{g}'(\underline{Z}) \underline{Q} \underline{g}(\underline{Z}) + \underline{U}'(t) \underline{R} \underline{U}(t) + \underline{\lambda}'[\underline{g}(\underline{Z}) + \underline{B}\underline{U} + \underline{W}_1 \ddot{X}_0(t) - \dot{\underline{Z}}] \tag{3.2}$$

The necessary conditions for the optimal solution are

$$\frac{\partial H}{\partial \underline{\lambda}} = 0 \quad ; \quad \frac{\partial H}{\partial \underline{U}} = 0 \quad ; \quad \frac{\partial H}{\partial \underline{Z}} + \dot{\underline{\lambda}}' = 0 \tag{3.3}$$

The first condition above leads to the state equation of motion given by Eq. (2.2). Substitution of Eq. (3.2) into the condition $\partial H/\partial \underline{U}=0$, yields

$$\underline{U}(t) = -0.5 \underline{R}^{-1} \underline{B}' \underline{\lambda} \tag{3.4}$$

Substituting Eq. (3.2) into the third condition,

$$\frac{\partial H}{\partial \underline{Z}} + \dot{\underline{\lambda}}' = 0 \tag{3.5}$$

one obtains

$$2\underline{\Delta}'(\underline{Z})\underline{Q}\underline{g}(\underline{Z}) + \underline{\Delta}'(\underline{Z})\underline{\lambda} + \dot{\underline{\lambda}} = 0 \tag{3.6}$$

in which $\underline{\Delta}(\underline{Z})$ is the derivative matrix

$$\underline{\Delta}(\underline{Z}) = \partial\underline{g}(\underline{Z})/\partial\underline{Z} \tag{3.7}$$

The first nonlinear control law is obtained by setting

$$\underline{\lambda} = \underline{P}\underline{g}(\underline{Z}) \tag{3.8}$$

in which $\underline{P}$ is a (2nx2n) matrix to be determined. Substitution of Eq. (3.8) into Eq. (3.4) leads to the following control law

$$\underline{U}(t) = -0.5\underline{R}^{-1}\underline{B}'\underline{P}\underline{g}(\underline{Z}) \tag{3.9}$$

Substituting Eq. (3.8) into Eq. (3.6), using the matrix equation of motion for $\dot{\underline{Z}}(t)$ and neglecting the external load $\ddot{X}_0(t)$, one obtains

$$\dot{\underline{P}} + \underline{\Delta}'(\underline{Z})\underline{P} + \underline{P}\underline{\Delta}(\underline{Z}) - 0.5\underline{P}\underline{\Delta}(\underline{Z})\underline{B}\underline{R}^{-1}\underline{B}'\underline{P} = -2\underline{\Delta}'(\underline{Z})\underline{Q} \tag{3.10}$$

Again, an equivalent linearization technique is used for the determination of the $\underline{P}$ matrix. The nonlinear structure is linearized at the initial equilibrium point $\underline{Z}=\underline{0}$ that is stable, i.e.,

$$\underline{\Delta}(\underline{Z}) \cong \underline{\Delta}(\underline{Z})|_{\underline{Z}=\underline{0}} = \underline{\Delta}_0 \tag{3.11}$$

and the transient part of the $\underline{P}$ matrix is neglected, i.e., $\dot{\underline{P}}=0$. Then, Eq. (3.10) becomes

$$\underline{\Delta}_0'\underline{P} + \underline{P}\underline{\Delta}_0 - 0.5\underline{P}\underline{\Delta}_0\underline{B}\underline{R}^{-1}\underline{B}'\underline{P} = -2\underline{\Delta}_0'\underline{Q} \tag{3.12}$$

To facilitate the solution of the constant matrix $\underline{P}$, the following transformation is made,

$$\underline{P} = \underline{\Delta}_0'\underline{P}_1 \tag{3.13}$$

in which $\underline{P}_1$ is a (2nx2n) constant matrix to be determined. Substituting Eq. (3.13) into Eq. (3.12) and premultiplying the resulting equation by $(\underline{\Delta}_0')^{-1}$, one obtains

$$\underline{\Delta}_0'\underline{P}_1 + \underline{P}_1\underline{\Delta}_0 - 0.5\underline{P}_1\underline{\Delta}_0\underline{B}\underline{R}^{-1}\underline{B}'\underline{\Delta}_0'\underline{P}_1 = -2\underline{Q} \tag{3.14}$$

Equation (3.14) is the matrix Riccati equation from which the Riccati matrix, $\underline{P}_1$, can be determined.

Thus, the control vector given by Eq. (3.9) becomes

$$\underline{U}(t) = -0.5\underline{R}^{-1}\underline{B}'\underline{\Delta}_0'\underline{P}_1\underline{g}(\underline{Z}) \tag{3.15}$$

in which $\underline{P}_1$ is the (2nx2n) Riccati matrix to be computed from Eq. (3.14).

The nonlinear control law derived above, Eqs. (3.14) and (3.15), is referred to as the first nonlinear control law. The second nonlinear control law is obtained by setting

$$\underline{\lambda} = \underline{\Delta}'(\underline{Z})\underline{P}\underline{g}(\underline{Z}) \tag{3.16}$$

Substitution of Eq. (3.16) into Eq. (3.4) leads to the control vector $\underline{U}(t)$ as follows

$$\underline{U}(t) = -0.5\underline{R}^{-1}\underline{B}'\underline{\Delta}'(\underline{Z})\underline{P}\underline{g}(\underline{Z}) \tag{3.17}$$

The condition for determining the $\underline{P}$ matrix is obtained by substituting Eq. (3.16) into Eq. (3.6), using the matrix equation of motion, Eq. (2.2), for $\dot{\underline{Z}}(t)$ and neglecting $\ddot{X}_0(t)$; with the result,

$$\dot{\underline{\Delta}}'(\underline{Z})\underline{P} + \underline{\Delta}'(\underline{Z})[\dot{\underline{P}} + \underline{\Delta}'(\underline{Z})\underline{P} + \underline{P}\underline{\Delta}(\underline{Z}) - 0.5\underline{P}\underline{\Delta}(\underline{Z})\underline{B}\underline{R}^{-1}\underline{B}'\underline{\Delta}'(\underline{Z})\underline{P} + 2\underline{Q}] = 0 \tag{3.18}$$

Again, an equivalent linearization at the initial equilibrium point $\underline{Z}=\underline{0}$, Eq. (3.11), is used such that $\dot{\underline{\Delta}}_0=0$, and the transient part of the $\underline{P}$ matrix is neglected, i.e., $\dot{\underline{P}}=\underline{0}$. Then Eq. (3.18) becomes

$$\underline{\Delta}_0'\underline{P} + \underline{P}\underline{\Delta}_0 - 0.5\underline{P}\underline{\Delta}_0\underline{B}\underline{R}^{-1}\underline{B}'\underline{\Delta}_0'\underline{P} = -2\underline{Q} \tag{3.19}$$

which is exactly the matrix Riccati equation. It can easily be observed that $\underline{P}_1$ in Eq. (3.14) is identical to $\underline{P}$ in Eq. (3.19). Thus, the first nonlinear control law, Eq. (3.15), is a special case of the second nonlinear control law, Eq. (3.17), in which $\underline{\Delta}'(\underline{Z})$ is replaced by the constant matrix $\underline{\Delta}_0$.

It should be mentioned that the control laws proposed in Eqs. (3.15) and (3.17) are nonlinear, because $\underline{g}(\underline{Z})$ is a nonlinear function of the state vector $\underline{Z}$ given by Eq. (2.3).

SECTION 4

GENERALIZED NONLINEAR CONTROL

In order to protect the equipments housed in the building, the absolute acceleration of the floor response must be reduced to an acceptable level. This can be accomplished by including the acceleration response in the performance index. A generalized performance index is proposed in the following

$$J = \int_0^{t_f} \left[g'(t) Q g(t) + \ddot{X}_a'(t) Q_a \ddot{X}_a(t) + U'(t) R U(t) \right] dt \tag{4.1}$$

in which $\underline{Q}_a$ is an (nxn) symmetric positive semidefinite weighting matrix and $\ddot{\underline{X}}_a(t)$ is the absolute acceleration vector for all floors. It follows from the matrix equation of motion, Eq. (2.1), that the absolute acceleration, $\ddot{\underline{X}}_a(t)$, can be expressed as

$$\begin{aligned} \ddot{X}_a(t) &= -M_0^{-1}[E_D(\dot{X}) + E_s(X)] + M_0^{-1} H U(t) \\ &= -L M^{-1}[E_D(\dot{X}) + E_s(X)] + L M^{-1} H U(t) \end{aligned} \tag{4.2}$$

in which $\underline{M}_0$ is a diagonal mass matrix with m_i being the ith diagonal element, and

$$L = M_0^{-1} M \tag{4.3}$$

is an (nxn) transformation matrix with $L(i,j)=1$ for $j \le i$ and $L(i,j)=0$ for $j>i$.

Substituting Eq. (4.2) into Eq. (4.1) and rearranging, one obtains

$$J = \int_0^{t_f} [g'(Z), U'(t)] \; T \begin{bmatrix} g(Z) \\ U(t) \end{bmatrix} dt \tag{4.4}$$

in which $\underline{T}$ is a (2n+r) x (2n+r) generalized weighting matrix

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{12}' & T_{22} \end{bmatrix} \tag{4.5}$$

In Eq. (4.5), $\underline{T}_{11}$ is a (2nx2n) matrix, $\underline{T}_{12}$ is a (2nxr) matrix and $\underline{T}_{22}$ is a (rxr) matrix given in the following

$$T_{11} = Q + \begin{bmatrix} 0_{nn} & 0_{nn} \\ 0_{nn} & T_a \end{bmatrix} \quad ; \quad T_{12} = \begin{bmatrix} 0_{nr} \\ T_a M^{-1} H \end{bmatrix} \tag{4.6}$$

$$T_{22} = R + (M^{-1}H)' T_a (M^{-1}H)$$

where T_a is a (nxn) transformed matrix of Q_a, i.e.,

$$T_a = L' Q_a L \tag{4.7}$$

To minimize the performance index given by Eq. (4.4), the Hamiltonian H is formed as follows

$$H = [g', U'] \, T \begin{bmatrix} g \\ U \end{bmatrix} + \lambda' [g + BU + W_1 \ddot{X}_0 - \dot{Z}] \tag{4.8}$$

in which the arguments Z and t have been dropped for simplicity.

The necessary conditions for the optimal solution are

$$\frac{\partial H}{\partial \lambda} = 0 \quad ; \quad \frac{\partial H}{\partial U} = 0 \quad ; \quad \frac{\partial H}{\partial Z} + \dot{\lambda}' = 0 \tag{4.9}$$

The first condition $\partial H/\partial \lambda$ leads to the state equation of motion given by Eq. (2.2). Substitution of Eq. (4.8) into the second and third conditions of Eq. (4.9) leads to the following relations

$$U(t) = -0.5 T_{22}^{-1} [B' \lambda + 2 T_{12}' g(Z)] \tag{4.10}$$

$$2\Delta'(Z) T_{11} g(Z) + 2\Delta'(Z) T_{12} U(t) + \Delta'(Z) \lambda + \dot{\lambda} = 0 \tag{4.11}$$

in which

$$\Delta(Z) = \partial g(Z) / \partial Z \tag{4.12}$$

is the system derivative matrix identical to Eq. (3.7).

Let

$$\lambda = \Delta'(Z) P g(Z) \tag{4.13}$$

Substituting Eq. (4.13) into Eqs. (4.10) and (4.11), one obtains

$$\underline{U}(t) = -0.5\underline{T}_{22}^{-1}[\underline{B}'\Delta'(\underline{Z})\underline{P} + 2\underline{T}_{12}']\underline{g}(\underline{Z}) \tag{4.14}$$

$$\dot{\Delta}'(\underline{Z})\underline{P}\underline{g}(\underline{Z}) + \Delta'(\underline{Z})[\underline{\dot{P}} + \bar{\Delta}'(\underline{Z})\underline{P} + \underline{P}\bar{\Delta}(\underline{Z}) - 0.5\underline{P}\Delta(\underline{Z})\underline{B}\underline{T}_{22}^{-1}\underline{B}'\Delta'(\underline{Z})\underline{P} + 2(\underline{T}_{11} - \underline{T}_{12}\underline{T}_{22}^{-1}\underline{T}_{12}')]\underline{g}(\underline{Z}) = 0 \tag{4.15}$$

in which

$$\bar{\Delta}(\underline{Z}) = \Delta(\underline{Z})[\underline{I} - \underline{B}\underline{T}_{22}^{-1}\underline{T}_{12}'] \tag{4.16}$$

At this point it is necessary to linearize the system in order to obtain a constant $\underline{P}$ matrix. Again, we linearize $\underline{\Delta}(\underline{Z})$ at the initial equilibrium point $\underline{Z}(t)=\underline{0}$ such that

$$\Delta(\underline{Z}) = \Delta_0 \textit{ and } \dot{\Delta}(\underline{Z}) = 0 \tag{4.17}$$

and neglect the transient part $\underline{\dot{P}}$. Then, Eq. (4.15) becomes

$$\bar{\Delta}_0'\underline{P} + \underline{P}\bar{\Delta}_0 - 0.5\underline{P}\Delta_0\underline{B}\underline{T}_{22}^{-1}\underline{B}'\Delta_0'\underline{P} = -2(\underline{T}_{11} - \underline{T}_{12}\underline{T}_{22}^{-1}\underline{T}_{12}') \tag{4.18}$$

in which $\bar{\underline{\Delta}}_0$ follows from Eqs. (4.16) and (4.17) as

$$\bar{\Delta}_0 = \Delta_0[\underline{I} - \underline{B}\underline{T}_{22}^{-1}\underline{T}_{12}'] \tag{4.19}$$

Equation (4.18) is the matrix Riccati equation from which the constant Riccati matrix $\underline{P}$ can be determined.

The control law presented in Eqs. (4.14) and (4.19) corresponds to the second nonlinear control law presented in the previous section. It can be shown easily that $\underline{T}_a=\underline{0}$ and $\underline{T}_{12}=\underline{0}$ for $\underline{Q}_a=\underline{0}$. Then, Eqs. (4.14) and (4.19) reduce to Eqs. (3.17) and (3.19), respectively. Furthermore, if $\underline{\Delta}(\underline{Z})$ appearing in Eq. (4.14) is linearized by $\underline{\Delta}_0$, one obtains

$$\underline{U}(t) = -0.5\underline{T}_{22}^{-1}[\underline{B}'\underline{\Delta}_0'\underline{P} + 2\underline{T}_{12}']\underline{g}(\underline{Z}) \tag{4.20}$$

which is the first nonlinear control law.

SECTION 5

GENERALIZED NONLINEAR CONTROL INCLUDING ACTUATOR DYNAMICS

In this section, the actuator dynamics will be taken into account in the derivation of the generalized nonlinear control law. For simplicity, the dynamic equations for r actuators are described by a system of first order differential equations [23]

$$\dot{\underline{U}}(t) + \underline{a}\underline{U}(t) = \underline{b}\underline{q}(t) \tag{5.1}$$

in which $\underline{a}$ and $\underline{b}$ are (rxr) diagonal matrices with diagonal elements a_i and b_i ($i=1,2,\ldots,r$). a_i is a measure of the loop gain or the reaction time of the ith actuator, and b_i is a measure of feedback gain or the amplification factor of the ith actuator. In Eq. (5.1), $\underline{q}(t)$ is a r-vector representing the feedback signal (or dynamic input) for generating the required active control vector $\underline{U}(t)$. Note that the vector $\underline{q}(t)$ is proportional to the control vector $\underline{U}(t)$ and it will be determined later through the optimization process. It should be mentioned that the extension of the optimal control theory in the following is not restricted to the first order differential equation, Eq. (5.1), for the actuator dynamics. Other higher order differential equations can similarly be used.

The dynamic equations of actuators, Eq. (5.1), can be augmented to the state equation of motion, Eq. (2.2), and both can be casted into the following (2n+r) vector equation

$$\dot{\underline{Z}}_1(t) = \underline{S}\,\underline{h}(\underline{Z}_1) + \bar{\underline{B}}\underline{q}(t) + \bar{\underline{W}}\ddot{X}_0(t) \tag{5.2}$$

in which

$$\underline{Z}_1(t) = \begin{bmatrix} \underline{Z}(t) \\ --- \\ \underline{U}(t) \end{bmatrix};\ \underline{h}(\underline{Z}_1) = \begin{bmatrix} \underline{g}(\underline{Z}) \\ --- \\ \underline{U}(t) \end{bmatrix};\ \underline{S} = \begin{bmatrix} \underline{I} & | & \underline{B} \\ ---- & | & ---- \\ \underline{0}_{r2n} & | & -\underline{a} \end{bmatrix}$$

$$\bar{\underline{B}} = \begin{bmatrix} \underline{0}_{2nr} \\ --- \\ \underline{b} \end{bmatrix};\ \bar{\underline{W}} = \begin{bmatrix} \underline{W}_1 \\ --- \\ \underline{0}_{r1} \end{bmatrix} \tag{5.3}$$

where $\underline{0}_{2nr}$, and $\underline{0}_{r1}$ are (2nxr) and (rx1) zero matrices, respectively.

The generalized performance index J can be expressed as

$$J = \int_0^{t_f}\left[\underline{g}'(Z)\underline{Q}\underline{g}(Z) + \underline{U}'(t)\underline{R}\underline{U}(t) + \ddot{\underline{X}}_a'(t)\underline{Q}_a\ddot{\underline{X}}_a(t) + \underline{q}'(t)\bar{\underline{R}}\underline{q}(t)\right]dt \tag{5.4}$$

in which $\underline{Q}_a$ is a (nxn) symmetric positive semidefinite weighting matrix, $\bar{\underline{R}}$ is a (rxr) symmetric positive definite weighting matrix, and $\underline{Q}$ and $\underline{R}$ have been defined previously.

In Eq. (5.4), $\ddot{\underline{X}}_a(t)$ is the absolute acceleration vector given by Eq. (4.2)

$$\ddot{\underline{X}}_a(t) = -\underline{L}\,\underline{M}^{-1}[\,\underline{F}_D(\dot{X}) + \underline{F}_S(X)\,] + \underline{L}\,\underline{M}^{-1}\underline{H}\,\underline{U}(t) \tag{5.5}$$

Substituting Eqs. (5.5) into Eq. (5.4), one obtains the following equivalent generalized performance index

$$J = \int_0^{t_f}\left[\underline{h}'(Z_1)\,\underline{T}\,\underline{h}(Z_1) + \underline{q}'(t)\bar{\underline{R}}\underline{q}(t)\right]dt \tag{5.6}$$

in which the $\underline{T}$ matrix is defined in Eqs. (4.5) to (4.7).

Following the same optimization procedures described in the previous sections, the optimal solution is obtained as

$$\underline{q}(t) = -0.5\bar{\underline{R}}^{-1}\,\bar{\underline{B}}'\,\hat{\Lambda}'(\underline{Z}_1)\,\hat{\underline{P}}\,\underline{h}(\underline{Z}_1) \tag{5.7}$$

in which $\hat{\underline{\Lambda}}(\underline{Z}_1)$ is the derivative matrix of $\underline{h}(\underline{Z}_1)$ with respect to $\underline{Z}_1$,

$$\hat{\Lambda}(Z_1) = \frac{\partial \underline{h}(\underline{Z}_1)}{\partial \underline{Z}_1} \tag{5.8}$$

The condition for determining the $\hat{\underline{P}}$ matrix is as follows

$$\dot{\hat{\Lambda}}'(Z_1)\hat{\underline{P}} + \hat{\Lambda}'(Z_1)[\dot{\hat{\underline{P}}} + \underline{S}'\hat{\Lambda}'(Z_1)\hat{\underline{P}} + \hat{\underline{P}}\,\hat{\Lambda}(Z_1)\underline{S}$$
$$- 0.5\,\hat{\underline{P}}\hat{\Lambda}(Z_1)\bar{\underline{B}}\bar{\underline{R}}^{-1}\bar{\underline{B}}'\hat{\Lambda}'(Z_1)\hat{\underline{P}} + 2\underline{T}] = 0 \tag{5.9}$$

Again, an equivalent linearization at the initial equilibrium point $\underline{Z}_1=0$ is used such that $\dot{\hat{\underline{\Lambda}}}_0=0$, and the transient part of the $\hat{\underline{P}}$ matrix is neglected, i.e., $\dot{\hat{\underline{P}}}=\underline{0}$. Then Eq. (5.9) becomes

$$(\hat{\Lambda}_0\underline{S})'\hat{\underline{P}} + \hat{\underline{P}}(\hat{\Lambda}_0\underline{S}) - 0.5\hat{\underline{P}}\hat{\Lambda}_0\bar{\underline{B}}\bar{\underline{R}}^{-1}\bar{\underline{B}}'\hat{\Lambda}_0'\hat{\underline{P}} = -2\underline{T} \tag{5.10}$$

which is exactly the matrix Riccati equation and

$$\hat{\Lambda}_0 = \hat{\Lambda}(\underline{Z}_1)|_{Z_1 = \underline{0}} \tag{5.11}$$

Equation (5.7) is the second optimal nonlinear control law. The first optimal nonlinear control law is identical to Eq. (5.7) except that $\hat{\underline{\Lambda}}'(\underline{Z}_1)$ is replaced by $\hat{\underline{\Lambda}}_0'$.

SECTION 6
SIMULATION OF STRUCTURAL RESPONSE

In order to evaluate the effectiveness and performance of the proposed optimal nonlinear control method, it is necessary to simulate the response of the controlled structure. A method of simulation for hysteretic structures is presented in the following. For simplicity, the damping of the structure is considered as linear viscous damping, i.e., $\underline{F}_D[\dot{\underline{X}}(t)] = \underline{C}\,\dot{\underline{X}}$, where $\underline{C}$ is the damping matrix.

The following hysteretic model will be used for both the structures and passive protective systems. The stiffness restoring force, $F_{si}(t)$, of the ith story unit is given by

$$F_{si}(t) = \alpha_i k_i x_i + (1-\alpha_i) k_i D_{yi} v_i \tag{6.1}$$

in which k_i = elastic stiffness of the ith story unit, α_i = ratio of the post-yielding to pre-yielding stiffness, D_{yi} = yield deformation = constant, and v_i is a nondimensional variable introduced to describe the hysteretic component of the deformation, with $|v_i| \le 1$, where

$$\dot{v}_i = D_{yi}^{-1}[A_i \dot{x}_i - \beta_i |\dot{x}_i|\,|v_i|^{n_i-1} v_i - \gamma_i \dot{x}_i |v_i|^{n_i}] = f_i(\dot{x}_i, v_i) \tag{6.2}$$

In Eq. (6.2), parameters A_i, β_i and γ_i govern the scale and general shape of the hysteresis loop, whereas the smoothness of the force-deformation curve is determined by the parameter n_i.

The state equation of the motion, Eq. (2.1), can be expressed as

$$\underline{M}\ddot{\underline{X}}(t) + \underline{C}\dot{\underline{X}}(t) + \underline{K}_e \underline{X}(t) + \underline{K}_I \underline{V}(t) = \underline{\xi}\ddot{X}_0(t) + \underline{H}_1 \underline{U}(t) \tag{6.3}$$

in which $\underline{V}(t) = [v_1(t), v_2(t), \ldots, v_n(t)]'$ = an n vector denoting the hysteretic component v_i of each story unit given by Eq. (6.2). In Eq. (6.3), $\underline{C}$, $\underline{K}_e$ and $\underline{K}_I$ are (nxn) band-limited damping matrix, elastic stiffness matrix and hysteretic stiffness matrix, respectively. All elements of $\underline{C}$, $\underline{K}_e$ and $\underline{K}_I$ are zero, except $C(i,i)=c_i$, $K_e(i,i) = \alpha_i k_i$, $K_I(i,i)=(1-\alpha_i)k_i D_{yi}$ for $i=1,2,\ldots,n$ and $C(i,i+1)=-c_{i+1}$, $K_e(i,i+1)=-\alpha_{i+1}k_{i+1}$, $k_I(i,i+1)=-(1-\alpha_{i+1})k_{i+1}D_{yi+1}$ for $i=1,2,\ldots,n-1$, where c_i is the damping coefficient of the ith story unit. The expressions given above for matrices $\underline{C}$, $\underline{K}_e$ and $\underline{K}_I$ hold for a base-isolated building connected to an actuator at the base isolation system, Fig. 7-1(a). When the arrangement of the control system is different, the matrices $\underline{C}$, $\underline{K}_e$ and $\underline{K}_I$ should be modified appropriately.

By introducing a 3n state vector $\underline{\tilde{Z}}(t)$, a (3nxr) matrix $\underline{\tilde{B}}$ and a 3n vector $\underline{\tilde{W}}_1$

$$\underline{\tilde{Z}}(t) = \begin{bmatrix} \underline{X} \\ \underline{Y} \\ \underline{\dot{X}} \end{bmatrix} \quad ; \quad \underline{\tilde{B}} = \begin{bmatrix} \underline{0} \\ \underline{0} \\ \underline{M}^{-1}\underline{H}_1 \end{bmatrix} \quad ; \quad \underline{\tilde{W}}_1 = \begin{bmatrix} \underline{0} \\ \underline{0} \\ \underline{M}^{-1}\underline{\xi} \end{bmatrix} \tag{6.4}$$

the second-order nonlinear vector equation of motion, Eq. (6.3), can be converted into a first order vector equation as follows

$$\underline{\dot{\tilde{Z}}}(t) = \underline{\tilde{g}}[\underline{\tilde{Z}}(t)] + \underline{\tilde{B}}\underline{U}(t) + \underline{\tilde{W}}_1\ddot{X}_0(t) \tag{6.5}$$

in which $\underline{\tilde{g}}[\underline{\tilde{Z}}(t)]$ is a 3n vector consisting of nonlinear functions of components of $\underline{\tilde{Z}}(t)$,

$$\underline{\tilde{g}}[\underline{\tilde{Z}}(t)] = \begin{bmatrix} \underline{\dot{X}} \\ ------------ \\ \underline{f}(\underline{\dot{X}},\underline{Y}) \\ ------------ \\ -\underline{M}^{-1}\left(\underline{C}\underline{\dot{X}} + \underline{K}_e\underline{X} + \underline{K}_I\underline{Y}\right) \end{bmatrix} \tag{6.6}$$

where $\underline{f}(\underline{\dot{X}},\underline{Y}) = [f_1(\dot{x}_1,v_1), f_2(\dot{x}_2,v_2),\ldots,f_n(\dot{x}_n,v_n)]'$ = an n vector with the ith element, $f_i(\dot{x}_i,v_i)$, given by Eq. (6.2).

The vector equation of motion given in Eq. (6.5) can be augmented by the actuator dynamics, Eq. (5.1), as follows

$$\underline{\dot{Z}}_2(t) = \underline{\tilde{S}}\underline{\tilde{h}}(\underline{Z}_2) + \underline{B}^*\underline{q}(t) + \underline{W}^*\ddot{X}_0(t) \tag{6.7}$$

in which $\underline{Z}_2(t)$, $\underline{\tilde{h}}(\underline{Z}_2)$ and W^* are (3n+r) vectors, $\underline{\tilde{S}}$ is a (3n+r)x(3n+r) matrix and $\underline{B}^*$ is a (3n+r)x(r) matrix,

$$\underline{Z}_2(t) = \begin{bmatrix} \underline{\tilde{Z}}(t) \\ --- \\ \underline{U}(t) \end{bmatrix} ; \ \underline{\tilde{h}}(\underline{Z}_2) = \begin{bmatrix} \underline{\tilde{g}}(\underline{\tilde{Z}}) \\ --- \\ \underline{U}(t) \end{bmatrix} ; \ \underline{\tilde{S}} = \begin{bmatrix} \underline{I} & | & \underline{\tilde{B}} \\ ---- & | & ---- \\ \underline{0}_{r2n} & | & -\underline{a} \end{bmatrix}$$

$$\underline{B}^* = \begin{bmatrix} \underline{0} \\ --- \\ \underline{b} \end{bmatrix} ; \ \underline{W}^* = \begin{bmatrix} \underline{\tilde{W}}_1 \\ --- \\ \underline{0} \end{bmatrix} \tag{6.8}$$

With the optimal nonlinear control law, $\underline{q}(t)$, derived in Eq. (5.7), the response for the hysteretic structural system can be simulated by solving Eq. (6.7) numerically using the Fourth-Order Runge-Kutta method [e.g., 18-21].

The derivative matrix $\hat{\underline{\Lambda}}(\underline{Z}_1)$ appearing in the control law, Eq. (5.7), is given by

$$\hat{\Lambda}(Z_1) = \begin{bmatrix} \Lambda(Z) & | & \underline{0}_{2nr} \\ ---- & | & ---- \\ \underline{0}_{r2n} & | & \underline{I}_{rr} \end{bmatrix} \tag{6.9}$$

in which $\underline{I}_{rr}$ = a (rxr) identity matrix and

$$\Lambda(Z) = \begin{bmatrix} \underline{0}_{nn} & | & \underline{I}_{nn} \\ ---------- & | & ---------- \\ -\underline{M}^{-1}[\underline{K}_e + \underline{K}_I \frac{\partial \underline{V}}{\partial \underline{X}}] & | & -\underline{M}^{-1}\underline{C} \end{bmatrix} \tag{6.10}$$

where $\partial\underline{V}/\partial\underline{X}$ is a diagonal matrix with the i diagonal element $\partial v_i/\partial x_i$ given as follows

$$\frac{\partial v_i}{\partial x_i} = \frac{\partial f_i(\dot{x}_i, v_i)}{\partial x_i} = D_{yi}^{-1}[A_i - \beta_i sgn(\dot{x}_i)\,|v_i|^{n_i-1} v_i - \gamma_i |v_i|^{n_i}] \tag{6.11}$$

The constant derivative matrix $\hat{\underline{\Lambda}}_0$ is given also by Eq. (6.9) except that $\underline{\Lambda}(\underline{Z})$ is replaced by $\underline{\Lambda}_0$ where

$$\Lambda_0 = \begin{bmatrix} \underline{0} & | & \underline{I} \\ ---- & | & ---- \\ -\underline{M}^{-1}\underline{K} & | & -\underline{M}^{-1}\underline{C} \end{bmatrix} \tag{6.12}$$

SECTION 7

OPTIMAL CONTROL USING ACCELERATION AND VELOCITY FEEDBACKS

Both linear and nonlinear control laws presented previously require the feedback of the state vector $\underline{Z}(t)$ that should be either measured or estimated using an observer. Frequently, it may be easier to measure the acceleration response than the displacement response [e.g., 20, 21]. Control laws using the acceleration and velocity feedbacks were suggested in Refs. 20 and 21. These control laws will be derived in this section using the LQR type formulation.

Consider the following LQR type performance index

$$J = \int_0^{t_f} [\dot{Z}'(t)\underline{Q}^*\dot{Z}(t) + \underline{U}'(t)\underline{R}\underline{U}(t)]\,dt \tag{7.1}$$

in which $\dot{\underline{Z}}(t)$ is the time derivative of the state vector, which consists of the velocity and acceleration responses. To minimize the objective function J subjected to the constraint of the state equation of motion, Eq. (2.2), the Hamiltonian H is introduced

$$H = \dot{Z}'(t)\underline{Q}^*Z(t) + \underline{U}'(t)\underline{R}\underline{U}(t) + \lambda[\underline{g}(Z) + \underline{B}\underline{U}(t) + \underline{W}_1\ddot{X}_0(t) - \dot{Z}(t)] \tag{7.2}$$

The necessary conditions for the optimal solution are

$$\frac{\partial H}{\partial \lambda} = 0 \;\; ; \;\; \frac{\partial H}{\partial \underline{U}} = 0 \;\; ; \;\; \frac{\partial H}{\partial Z} - \frac{d}{dt}\left[\frac{\partial H}{\partial \dot{Z}}\right] = 0 \tag{7.3}$$

in which the general form has been used for the third condition. Substitution of Eq. (7.2) into the last two conditions yields

$$\underline{U}(t) = -0.5\underline{R}^{-1}\underline{B}'\lambda \tag{7.4}$$

$$\Lambda'(Z)\lambda + 2\underline{Q}^*\ddot{Z}(t) + \dot{\lambda} = 0 \tag{7.5}$$

in which $\underline{\Lambda}(\underline{Z})$ is the derivative matrix

$$\frac{\partial \underline{g}(Z)}{\partial Z} = \Lambda(Z) \tag{7.6}$$

At this point, we shall linearize the equation of motion at the initial equilibrium point $\underline{Z}=0$, such that

$$\Delta(Z) \approx \Delta(Z)\,|_{Z=0} = \Delta_0 \qquad (7.7)$$
$$g(Z) \approx \Delta_0 Z$$

Let

$$\lambda = -(\Delta_0^{-1})' P \dot{Z} \qquad (7.8)$$

in which P is a constant matrix to be determined. Substituting Eq. (7.8) into Eqs. (7.4) and (7.5) and neglecting the external excitation, one obtains

$$U(t) = 0.5 R^{-1} B' (\Delta_0^{-1})' P \dot{Z} \qquad (7.9)$$

$$-P\dot{Z} + 2Q^{*}\ddot{Z} - (\Delta_0^{-1})' P \ddot{Z} = 0 \qquad (7.10)$$

Substituting Eq. (7.9) into the linearized state equation of motion, taking the time derivative of the resulting equation, and substituting the resulting equation into Eq. (7.10), one obtains the following matrix Riccati equation for the determination of the P matrix,

$$P\Delta^{*} + (\Delta^{*})' P - 0.5 P B^{*} R^{-1} (B^{*})' P + 2Q^{*} = 0 \qquad (7.12)$$

in which

$$\Delta^{*} = \Delta_0^{-1} \quad ; \quad B^{*} = \Delta_0^{-1} B \qquad (7.13)$$

If the equation of motion is linear, i.e.,

$$g(Z) = AZ \, ; \, \Delta_0 = A \qquad (7.14)$$

then, the control law given by Eqs. (7.9) and (7.12) is the exact optimal control which was presented in Ref. 20. However, if the equation of motion is nonlinear, the control law given by Eqs. (7.9) and (7.12) is an approximation which was proposed in Ref. 21.

SECTION 8
NUMERICAL SIMULATION

To demonstrate the performance of the proposed nonlinear control method and to compare it with that of the linear control method, numerical examples are worked out in this section. Two cases are considered in the following; namely, a moderate earthquake (0.3g) and a strong earthquake (1g).

Example 1: A Base-Isolated Elasto-Plastic Building

An eight-story building that exhibits bilinear elasto-plastic behavior is considered, Fig. 7.1 [e.g., 18-20]. The properties of the building are as follows : (i) the mass of each floor is identical with m_i = m=345.6 metric tons; (ii) the preyielding stiffnesses of the eight-story units are k_{i1} (i=1,2,...,8)=3.4x10^5, 3.26x10^5, 2.85x10^5, 2.69x10^5, 2.43x10^5, 2.07x10^5, 1.69x10^5 and 1.37x10^5 kN/m, respectively, and the postyielding stiffnesses are $k_{i2}=0.1\ k_{i1}$ for i=1,2,...,8, i.e., $\alpha_i=0.1$ and $k_i=k_{i1}$; and (iii) the viscous damping coefficients for each story unit are c_i=490, 467, 410, 386, 348, 298, 243 and 196 kN.sec/m, respectively. The damping coefficients given above result in a damping ratio of 0.38% for the first vibrational model. The fundamental frequency of the unyielded building is 5.24 rad./sec. The yielding level for each story unit varies with respect to the stiffness; with the results, D_{yi}=2.4, 2.3, 2.2, 2.1, 2.0, 1.9, 1.7, and 1.5 cm. The bilinear elasto-plastic behavior can be described by the hysteretic model, Eqs. (6.1) and (6.2), with A_i=1.0, β_i=0.5, n_i=95 and γ_i=0.5 for i=1,2,...,8 [Ref. 18]. The same El Centro earthquake with a maximum ground acceleration of 0.3g as shown in Fig. 6.2 of Part I is used as the input excitation.

Time histories of all the response quantities have been computed. Within 30 seconds of the earthquake episode, the maximum interstory deformation, x_i, and the maximum absolute acceleration of each floor, $\ddot{x}_{ai}$, are shown in columns (3) and (4) of Table 7.1, designated as "No Control". As observed from Table 7.1 , the deformation of the unprotected building is excessive and that yielding takes place in the upper five story units.

To reduce the structural response, a lead-core rubber-bearing isolation system is implemented as shown in Fig. 7.1(a). The restoring force of the lead-core rubber-bearing isolation system is modeled by Eq. (6.1) with $F_{sb}=\alpha_b k_b x_b + (1-\alpha_b)k_b D_{yb} v_b$ in which v_b is given by Eq. (6.2) with i = b. The mass of the base isolation system is $m_b=450$ metric tons and the viscous damping coefficient is assumed to be linear with $c_b=26.17$ kN sec/m. The restoring force of the base isolation system given above is not bilinear elasto-plastic and the parameter values are given as follows: $k_b=18050$ kN/m, $\alpha_b=0.6$, $D_{yb}=4$cm, $A_b=1.0$, $\beta_b=0.5$, $n_b=3$ and $\gamma_b=0.5$, Eq. (6.2). The hysteresis loop of such a base isolation system, i.e., x_b versus v_b, is shown in Fig. 7.2. For the building with the base isolation system alone, the first natural frequency of the preyielded structure is 2.21 rad/sec and the damping ratio for the first vibrational mode is 0.15%. The response vector $\underline{X}(t)$ is given by $\underline{X}=[x_b,x_1,\ldots,x_8]'$.

The maximum response quantities of the building within 30 seconds of the earthquake episode are shown in columns (5) and (6) of Table 7.1 designated as "With BIS". As observed from Table 7.1, the interstory deformation and the floor acceleration are drastically reduced. However, the deformation of the base isolation system shown in row B of Table 7.1 should be reduced.

Since the effect of actuator dynamics has been demonstrated in Part I, it is not necessary to present similar results. It is mentioned that the degradation of the control performance due to the actuator response is minimal as long as the actuator dynamics is taken into account. In what follows, we shall assume that the time delay due to the actuator response is negligible, i.e., $\alpha=\beta$ is a large value, so that $\underline{q}(t)=\underline{U}(t)$.

To protect the safety and integrity of the base isolation system, an actuator is connected to the base isolation system as shown in Fig. 7.1(a). With the actuator applying the active control force $\underline{U}(t)$ to the base isolation system, the structural response depends on the weighting matrices $\underline{Q}$, $\underline{Q}_a$ and $\underline{R}$ where $\tilde{\underline{R}}=\underline{0}$. For this example, the weighting matrix $\underline{R}$ consists of only one element, denote by R_0.

We first consider the linear control law given by Eqs.(2.11) and (2.16). The (18x18) $\underline{Q}$ matrix is considered a diagonal matrix with all the diagonal elements equal to 1.0, i.e., Q(i,i)=1 for i=1, 2, ...,18, and $R_0=10^{-7}$ is used. Time histories of all the response quantities have been computed. Within 30 seconds of the earthquake episode, the maximum response quantities and the required maximum control force, U, are shown in columns (7) and (8) of Table 7.1. As observed from the table, the deformation of the base isolation system is reduced by 50%, where the response quantities of the superstructure reduce slightly except that of the top story unit.

The first nonlinear control law presented in Eq.(4.20) is considered in which the Riccati matrix $\underline{P}$ is computed from Eq.(4.18). In this case, the (18x18) diagonal $\underline{Q}$ matrix is assigned as follows : Q(i,i)=2 for i=1,2,...,9 and Q(i,i)=0 for i=10, 11, ..., 16. The (9x9) $\underline{Q}_a$ matrix that corresponds to the acceleration response is considered a diagonal matrix with all the diagonal elements equal to 1.0, i.e., $Q_a(i,i)=1$ for i=1,2,...,9, and $R_0=2\times10^{-7}$ is used. The maximum response quantities and the required maximum control force are presented in columns (9) and (10) of Table 7.1, designated as "Nonlinear Control I". It is observed that all the response quantities and the active control force are smaller than those associated with optimal linear control, columns (7) and (8). In particular, the acceleration response quantities improve significantly because of the use of the generalized performance index.

We next consider the second nonlinear control law in Eq. (4.14) and use the same matrices $\underline{Q}$, $\underline{Q}_a$ and $\underline{R}$ in the first nonlinear control law. The corresponding results are shown in columns (11) and (12) of Table 7.1, designated as "Nonlinear Control II'. It is observed from the table that all the response quantities and the required active control force are slightly smaller than those associated with the first nonlinear control law, columns (9) and (10). As a result, the performance of the second nonlinear control law is slightly better. It should be mentioned that for the second nonlinear control law the system derivative matrix, $\underline{\Lambda}(\underline{Z})$, is not linearized, Hence, the system derivative matrix $\underline{\Lambda}(\underline{Z})$ should be computed on-line, resulting in an increase of the on-line computational efforts.

Suppose the same base-isolated building used above is subjected to the El Centro earthquake shown in Fig. 6.2 of Part I but scaled uniformly to a maximum ground acceleration 1g. With such a strong earthquake input, the maximum response quantities of the building with and without a base isolation system are presented in columns (3)-(6) of Table 7.2, designated as "No Control" and "With BIS", respectively. It is observed that (i) Without a base isolation system, the ductility of the building response is quite high, and (ii)

With a base isolation system the response of the rubber bearing is too large, whereas the response quantities of all the story units are close to the yield limit D_y.

With the same active control devices and the same weighting matrices, $\underline{Q}$, $\underline{Q}_a$ and $\underline{R}$ used previously, the maximum response quantities for the corresponding control methods are presented in Table 7.2. The same conclusions described previously are obtained from Table 7.2: (i) the control performance of the two nonlinear control methods is better than that of the linear control method, and (ii) the control performance of the second nonlinear control method is slightly better than that of the first one.

Example 2: An Elasto-Plastic Building With Active Bracing Systems

The same eight-story elasto-plastic building considered in Example 1 is subjected to the same El Centro earthquake with a maximum ground acceleration of 1g. However, instead of using a rubber-bearing isolation system, an active bracing system is installed on every floor. The angle of inclination of the bracings with respect to the floor is 25°. Hence, the dimensions of the weighting matrices $\underline{Q}$, $\underline{Q}_a$ and $\underline{R}$ are (16x16), (8x8) and (8x8), respectively. These weighting matrices will be assigned as diagonal matrices in the following. Also, the time delay due to the actuator response is assumed to be negligible, i.e., $\alpha=\beta$ = a large value.

For the linear control law given by Eqs. (2.11) and (2.16), $Q(i,i)=1$ for $i=1,2,\ldots,16$ and $R(i,i)=10^{-9}$ for $i=1,2,\ldots,8$ are used. For nonlinear control laws, we choose $Q(1,1)=4000$, $Q(i,i)=1000$ for $i=2,3,\ldots,8$, $Q(i,i)=0$ for $i=9,10,\ldots,16$, and $Q_a(i,i)=1$, $R(i,i)=10^{-8}$ for $i=1,2,\ldots,8$.

Within 30 seconds of the earthquake episode, the maximum response quantities are presented in Table 7.3, including the maximum interstory deformation, x_i, the maximum absolute acceleration, $\ddot{x}_{ai}$, and the maximum control force U_i. It is observed from Table 7.3 that the control performance of all control laws is quite satisfactory. Further, the second nonlinear control law seems to perform slightly better than the first one.

The results shown in Table 7.3 indicate that yielding still occurs in some story units. To bring the response of each story unit into the elastic range, larger control forces are needed. For the linear control law, the $\underline{Q}$ matrix remains the same but each diagonal element of the $\underline{R}$ matrix is changed to 10^{-10}. For the nonlinear control laws, the $\underline{Q}_a$ and $\underline{R}$ matrices remain the same, but Q(1,1)=4000, Q(i,i)=1000 for i=2,3,...,8 and Q(i,i)=0 for i=9,10,...,16 are used. The corresponding maximum response quantities are presented in Table 7.4. Again, the control performance of all the control laws is very satisfactory. When all the response quantities are within the linear elastic range, there is no difference between the first and the second nonlinear control laws as evidenced by the results shown in Table 7.4.

Finally, we would like to point out that control of hysteretic buildings, in which a large ductility is involved, requires more controllers. For a perfectly linear elastic building, either an active mass damper installed on the top floor or an active bracing system (ABS) installed on the first floor is enough to control the entire building. However, for the elasto-plastic eight-story building subjected to a 1g earthquake considered in this example, either an active mass damper or an active bracing system alone is not capable of controlling the building response. The reason is that once a story unit yields with a large ductility, controllers installed in the other story unit cannot effectively exert the control force through the given load path. As a result, eight controllers are used in this example.

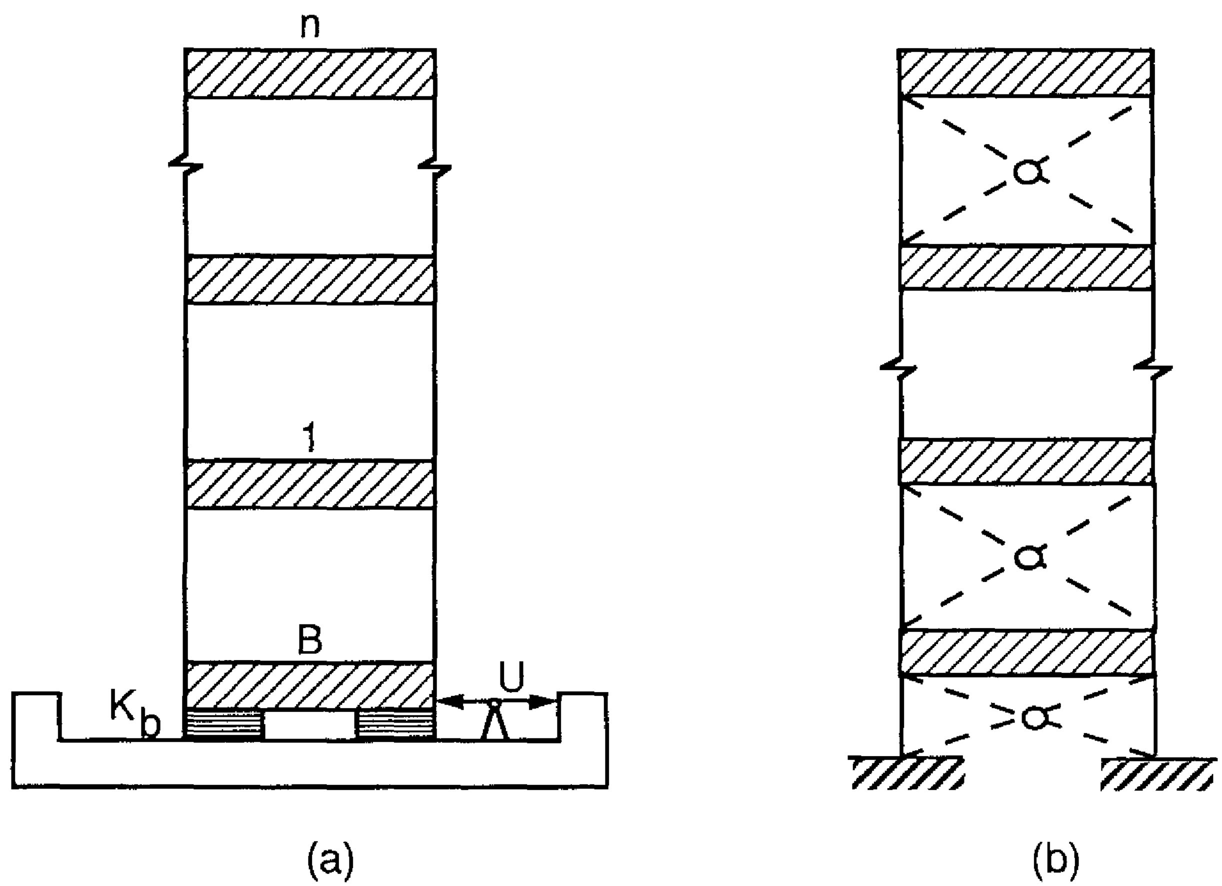

Fig. 7.1 : Structural Model of a Multi-Story Building : (a) With Rubber Bearing Isolation System and Actuator ; (b) With Active Bracing System

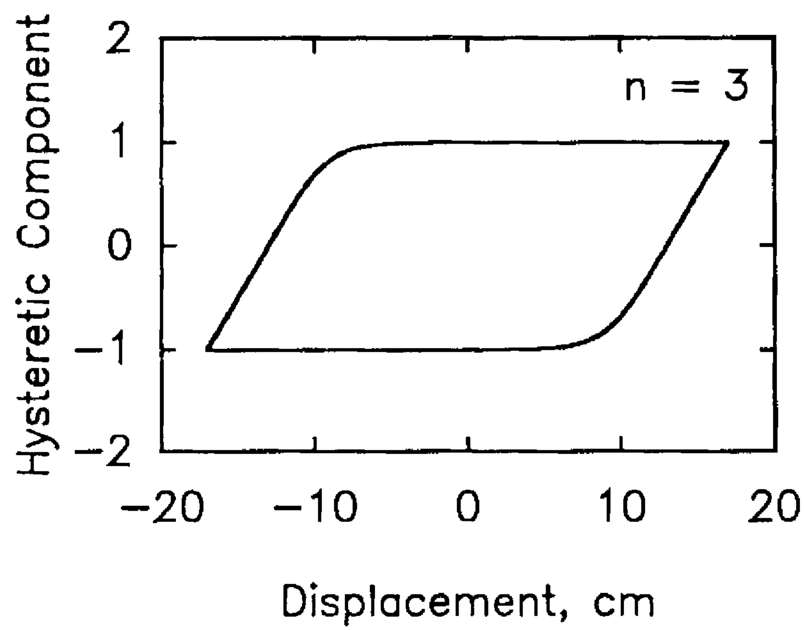

Fig. 7.2 : Hysteresis Loop of Lead–Core Rubber Bearing

Table 7.1 : Maximum Response Quantities of A Base-Isolated Building (0.3g Earthquake)

STORY NO		No Control		WITH BIS		LINEAR CONTROL U= 1106 kN		NONLINEAR CONTROL I U= 976 kN		NONLINEAR CONTROL II U= 951 kN	
(1)	D_y cm (2)	x_i cm (3)	$\ddot{x}_{ai}$ cm/s^2 (4)	x_i cm (5)	$\ddot{x}_{ai}$ cm/s^2 (6)	x_i cm (7)	$\ddot{x}_{ai}$ cm/s^2 (8)	x_i cm (9)	$\ddot{x}_{ai}$ cm/s^2 (10)	x_i cm (11)	$\ddot{x}_{ai}$ cm/s^2 (12)
B	-	-	-	21.36	117	10.14	124	8 31	65	8.06	63
1	2.4	2.05	381	0.62	112	0.51	124	0.38	65	0.37	63
2	2.3	2.08	434	0.59	112	0.52	115	0.38	61	0.37	59
3	2.2	2.16	500	0.65	111	0.60	98	0.40	58	0.39	57
4	2.1	2.40	460	0.63	101	0.61	86	0.38	52	0.36	50
5	2.0	2.68	559	0.63	91	0.58	101	0.35	62	0.34	60
6	1.9	3.16	446	0.64	103	0.56	105	0.34	64	0.33	62
7	1.7	4.37	594	0.60	130	0.59	120	0.32	70	0.31	67
8	1.5	1.94	614	0.41	163	0.46	184	0.22	88	0.22	85

Table 7.2 : Maximum Response Quantities of A Base-Isolated Building (1g Earthquake)

STORY NO		No Control		WITH BIS		LINEAR CONTROL U= 3874 kN		NONLINEAR CONTROL I U= 3247 kN		NONLINEAR CONTROL II U= 3109 kN	
(1)	D_y cm (2)	x_i cm (3)	$\ddot{x}_{ai}$ cm/s^2 (4)	x_i cm (5)	$\ddot{x}_{ai}$ cm/s^2 (6)	x_i cm (7)	$\ddot{x}_{ai}$ cm/s^2 (8)	x_i cm (9)	$\ddot{x}_{ai}$ cm/s^2 (10)	x_i cm (11)	$\ddot{x}_{ai}$ cm/s^2 (12)
B	-	-	-	67.61	351	29.69	316	25.97	212	24.95	231
1	2.4	5.04	1032	1.92	350	1.48	303	1.21	213	1.16	216
2	2.3	4.24	1150	1.95	325	1.44	312	1.21	200	1.16	181
3	2.2	5.31	1070	2.13	304	1.52	282	1.27	183	1.22	166
4	2.1	5.48	1171	2.07	277	1.48	255	1.19	165	1.15	147
5	2.0	6.76	1203	1.95	304	1.33	278	1.12	197	1.08	167
6	1.9	8.67	1000	1.81	345	1.28	348	1.06	208	1.01	184
7	1.7	10.36	805	1.59	374	1.44	356	0.99	219	0.96	222
8	1.5	4.62	726	1.07	421	1.14	449	0.75	298	0.72	285

Table 7.3 : Maximum Response Quantities of An Elasto-Plastic Building With Active Bracing System

STORY		No Control		Linear Control			1St Nonlinear Cntr			2ndNonlinear Cntr		
NO (1)	D_y cm (2)	x_i cm (3)	$\ddot{x}_{ai}$ cm/s² (4)	x_i cm (5)	$\ddot{x}_{ai}$ cm/s² (6)	U_i kN (7)	x_i cm (8)	$\ddot{x}_{ai}$ cm/s² (9)	U_i kN (10)	x_i cm (11)	$\ddot{x}_{ai}$ cm/s² (12)	U_i kN (13)
1	2.4	5.04	1032	3.33	838	9167	2.06	901	11605	2.05	897	11558
2	2.3	4.24	1151	3.18	717	8629	3.73	758	12152	3.71	755	12085
3	2.2	5.31	1070	3.34	621	8176	2.15	666	10230	2.14	664	10190
4	2.1	5.48	1172	2.80	543	7315	1.54	599	8518	1.53	597	8483
5	2.0	6.76	1204	2.24	494	6398	1.12	545	6727	1.11	543	6700
6	1.9	8.67	1001	1.85	529	5236	0.79	506	4934	0.79	504	4914
7	1.7	10.36	805	1.41	559	3761	0.52	479	3221	0.51	477	3208
8	1.5	4.62	726	0.79	576	1968	0.26	466	1588	0.26	464	1582

Table 7.4 : Maximum Response Quantities of An Elasto-Plastic Building With Active Bracing System (Large Control Force)

STORY		No Control		Linear Control			1St Nonlinear Cntr			2nd Nonlinear Cntr		
NO (1)	D_y cm (2)	x_i cm (3)	$\ddot{x}_{ai}$ cm/s² (4)	x_i cm (5)	$\ddot{x}_{ai}$ cm/s² (6)	U_i kN (7)	x_i cm (8)	$\ddot{x}_{ai}$ cm/s² (9)	U_i kN (10)	x_i cm (11)	$\ddot{x}_{ai}$ cm/s² (12)	U_i kN (13)
1	2.4	5.04	1032	1.72	912	18969	0.85	954	21589	0.85	954	21568
2	2.3	4.24	1151	1.54	853	16791	1.43	906	17929	1.43	905	17911
3	2.2	5.31	1070	1.39	811	14736	0.96	874	16010	0.96	873	15994
4	2.1	5.48	1172	1.19	792	12453	0.69	851	13498	0.69	851	13485
5	2.0	6.76	1204	0.99	790	10132	0.51	835	10871	0.51	834	10861
6	1.9	8.67	1001	0.78	789	7727	0.36	824	8197	0.36	824	8190
7	1.7	10.36	805	0.54	790	5222	0.23	818	5491	0.23	817	5486
8	1.5	4.62	726	0.28	791	2637	0.12	814	2756	0.11	814	2753

SECTION 9
CONCLUSIONS

An optimal nonlinear control method is proposed for applications to seismic-excited nonlinear or hysteretic building structures. Emphasis is placed on hybrid control of base-isolated hysteretic buildings. Both the absolute acceleration response of the building and the actuator dynamics have been accounted for in the optimization process. Control laws using acceleration and velocity feedbacks are also derived. Simulation results indicate that (i) the proposed nonlinear control method is effective for hybrid control of seismic-excited buildings isolated by rubber-bearing isolators, and (ii) the performance of the proposed nonlinear control method is better than that of the linear control method proposed previously.

SECTION 10

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