HAZARD AND RISK ASSESSMENT PROBLEMS

Assessing potential human health risks is a demanding task and one with far-reaching consequences. Risk assessment is a methodology that may be used for inferring the risk to public health from man-made substances in the environment. Estimating the occurrence of cancer as a result of exposure to various agents has been the primary focus of risk assessments.

This section of the problem workbook is concerned with quantitative methods involved in the analysis of hazardous conditions in terms of frequency of occurrence of unfavorable consequences. Uncertainty characterizes not only the possible transformation of a hazard into an accident, disaster or catastrophe, but also the effects of such a transformation. Measurement of uncertainty falls within the purview of mathematical probability. Accordingly, the problems present fundamental concepts and theorems of probability used in risk assessment, special probability distributions, and techniques pertinent to risk assessment.

HAZARD AND RISK ASSESSMENT 1

Conditional Probability

Introduction

The probability of event A, P(A), can be interpreted as a theoretical relative frequency, i.e., a number about which the relative frequency of event A tends to cluster as n, the number of times a random experiment (on the event) is performed, increases indefinitely. This is the objective interpretation of probability. Probability can also be interpreted subjectively as a measure of degree of belief, on a scale from 0 to 1, that the event A occurs. This interpretation is frequently used in ordinary conversation. For example, if someone says, "The probability I will go to the movies tonight is 90 %", then 90 % is a measure of the person's belief that he or she will go to the movies. This interpretation is also used when, in the absence of concrete data needed to estimate an unknown probability on the basis of observed relative frequency, the personal opinion of an expert is sought to provide the estimate.

The conditional probability of event B given A is denoted by $P(B|A)$ and defined as follows:

$$P(B|A) = P(AB)/P(A)$$

where: $P(AB)$ = probability events A and B occur.

$P(B|A)$ can be interpreted as the proportion of A occurrences that also feature the occurrence of B.

Problem Statement and Data

Two items are drawn in succession from a lot of 100 items, of which 10 are defective. What is the probability that both items are defective if a) the first is replaced before the second is drawn and b) the first is not replaced before the second is drawn?

Solution

1. First, determine the probability that the first item is defective, P(A).

Remember that 10 out of 100 are defective.

2. Determine the probability that the second item is defective if the first is replaced, P(B).

Since the first item is replaced, the probability for the defective item is the same.

3. Determine the probability that the two items are defective if the first is replaced, i.e., P(AB).

Note that P(AB) = P(A) P(B).

4. Determine the probability that the second item is defective if the first item is not replaced, i.e, P(B|A).

Remember that now the remaining lot contains 99 items, of which 9 are defective.

5. Determine the probability that the both items are defective if the first item is not replaced, i.e., P'(AB).

$P'(AB) = P(A)\ P(B|A)$

Comment

Conditional probability can be used to formulate a definition for the independence of two events A and B. Event B is defined to be independent of event A only if $P(B|A) = P(B)$. Similarly, event A is defined to be independent of event B if and only if $P(A|B) = P(A)$. From the definition of conditional probability, one can deduce the logically equivalent definition of the independence of event A and event B if and only if $P(AB) = P(A)\ P(B)$.

Bayes' Theorem

Introduction

Consider n mutually exclusive events $A_1, A_2, \ldots, A_n$. Let B be any given event. Then Bayes' Theorem states:

$$P(A_i|B) = P(A_i)\ P(B|A_i) / \sum_{i=1}^{n} P(A_i)\ P(B|A_i);\ i = 1, .., n$$

where: $P(A_i)$ = prior probability of A_i
$P(A_i|B)$ = posterior probability of A_i.

Bayes' Theorem provides the mechanism for revising prior probabilities, i.e., converting them into posterior probabilities on the basis of occurrence of some given event.

Problem Statement and Data

A casualty insurance company has low, medium, and high risk policy holders who have, respectively, probabilities of 0.0025, 0.01, and 0.01 of filing claims within a given year. The proportions of company policy holders in the three risk groups are 0.7, 0.2, and 0.1, respectively. What proportions of claims filed each year come from the low risk group of policy holders?

Solution

1. List the probabilities of the event of a policy holder being low risk, medium risk, and high risk.

Let A_1, A_2, and A_3 be the event of a policy holder being low risk, medium risk, and high risk, respectively.

$P(A_1)$ =
$P(A_2)$ =
$P(A_3)$ =

2. List the probabilities of the low, medium, and high risk policy holders filing claims within a given year.

Let B be the event of policy holder filing a claim.

$P(B|A_1)$ =
$P(B|A_2)$ =
$P(B|A_3)$ =

3. Apply Bayes' Theorem to obtain the probability of claims being filed by the low risk policy holder, $P(A_1|B)$.

Note that $P(A_1|B) = P(A_1)\ P(B|A_1)/[P(A_1)\ P(B|A_1) + P(A_2)\ P(B|A_2) + P(A_3)\ P(B|A_3)]$.

Comment

Bayes' Theorem is a simple, yet powerful, statistical tool that can be used for revising probabilities based on additional probability data. Note that the theorem may be applied to n mutually exclusive events, i.e., n may take on any positive integer value. It provides the mechanism for converting a prior probability distribution function (presented in next problem) to a posterior probability distribution function on the basis of the occurrence of another event (say B) or other events.

Probability Distribution Function

Introduction

The probability distribution of a random variable concerns the distribution of probability over the range of the random variable. The distribution of probability is specified by the probability distribution function (pdf). The random variable may be discrete or continuous. Special pdfs finding considerable application in risk analysis are considered in later problems. The pdf of a continuous random variable X has the following properties:

1. $\int_a^b f(x)\,dx = P(a < X < b)$
2. $f(x) >= 0$
3. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

where: $P(a < X < b)$ = probability assigned to an outcome or an event corresponding to the number x in the range of X between a and b

$f(x)$ = pdf of the continuous random variable X.

Property (1) indicates that the pdf of a continuous random variable generates probability by integration of the pdf over the interval whose probability is required. When this interval contracts to a single value, the integral over the interval becomes zero. Therefore, the probability associated with any particular value of a continuous random variable is zero. Consequently, if X is continuous,

$$P(a <= X <= b) = P(a < X <= b) = P(a < X < b)$$

Property (2) restricts the values of f(x) to non-negative numbers. Property (3) follows from the fact that:

$$P(-\infty < X < \infty) = 1$$

Problem Statement and Data

The difference between the magnitude of a large earthquake, as measured on the Richter scale, and the threshold value of 3.25, is a random variable X having the following probability distribution function (pdf):

$$f(x) = 1.7 \exp(-1.7x) \quad ; x > 0$$
$$f(x) = 0 \quad ; \text{elsewhere}$$

Find the probability that X will have a value between 2 and 6, i.e., $P(2 < X < 6)$ and find the variance of X.

Solution

1. Calculate the probability that X will have a value between 2 and 6.

Use property (1), i.e.,

$$\int_a^b f(x)\,dx = P(a < X < b)$$

Note that only f(x) for x >0 needs to be considered since the range is between 2 and 6.

2. Calculate the variance of X, σ^2.

By definition: $\sigma^2 = E(X^2) - (E(X))^2$

where: $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$E(X^2) = \int_{-\infty}^{\infty} x^2\, f(x)\, dx$$

a. In order to calculate the variance, the average value of X or the expected value of X, E(X) needs to be calculated first.

Note that the lower limit of the integral can begin at zero rather than negative infinity since f(x) = 0 at x<= 0. The expected value of X, E(X) is also called "the mean of X" and is often designated by the special symbol μ.

b. Calculate the value of $E(X^2)$.

c. Calculate the variance, σ^2.

Comment

The pdf of a discrete (rather than a continuous) random variable X is specified by f(x) where f(x) has the following essential properties:

1. $f(x) = P(X = x)$
   = probability assigned to the outcome corresponding to the number x in the range of X.

2. $f(x) >= 0$

3. $\sum_x f(x) = 1$

Property (1) indicates that the pdf of a discrete random variable generates probability by substitution. Properties (2) and (3) restrict the values of f(x) to non-negative real numbers whose sum is 1.

HAZARD AND RISK ASSESSMENT 4

Series and Parallel Systems

Introduction

Many systems consisting of several components can be classified as series, parallel, or a combination of both. A series system is one in which the entire system fails to operate if any one of its components fail to operate. If such a system consists of n components which function independently, then the reliability of the system is the product of the reliabilities of the individual components. If R_s denotes the reliability of a series system and R_i denotes the reliability of the ith component; i = 1, 2, ..., n; then

$$R_s = R_1 R_2 \ldots . R_n$$

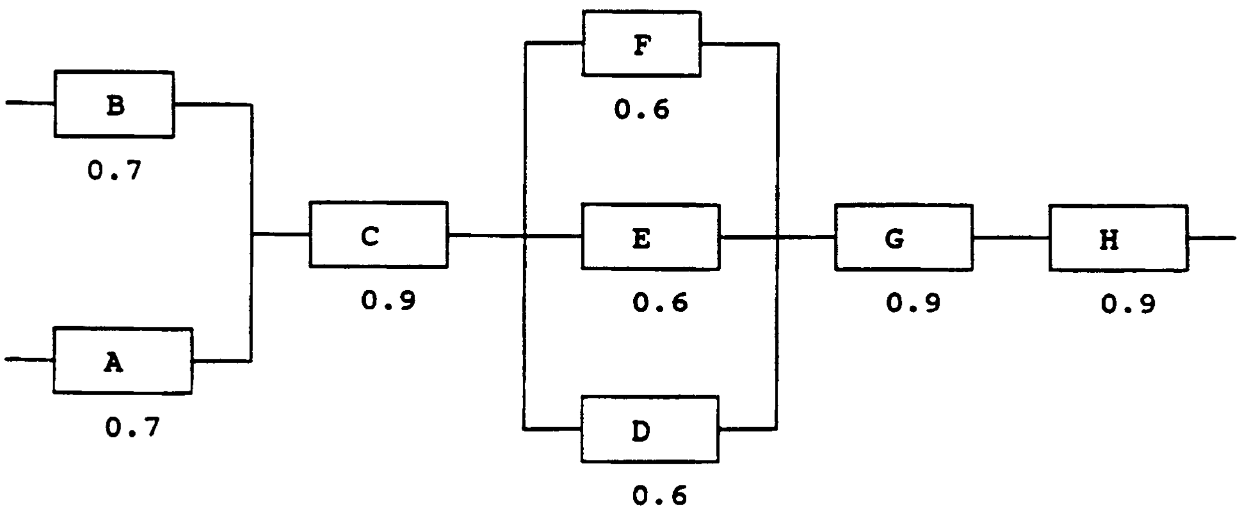
A parallel system is one which fails to operate only if all of its components fail to operate. If R_i is the reliability of the ith component, then $(1 - R_i)$ is the probability that the ith component fails. Assuming all n components function independently, the probability that all n components fail is $(1 - R_1)(1 - R_2) \ldots\ldots (1 - R_n)$. Subtracting this product from 1 yields the following formula for R_p, the reliability of a parallel system:

$$R_p = 1 - (1 - R_1)(1 - R_2) \ldots\ldots (1 - R_n)$$

The reliability formulas for series and parallel systems can be used to obtain the reliability of a system which combines features of a series and a parallel system.

Problem Statement and Data

Consider the electrical system diagrammed below with reliabilities indicated under the various components. Determine the reliability of the system.



Solution

1. Since the system is a combined system of both series and parallel systems, the parallel blocks should be combined into a single block first.

See the introductory section.

a. Combine blocks A and B, i.e., calculate the combined reliability of A and B, R_{AB}.

Use the equation for parallel system.

b. Calculate the combined reliability of D, E, and F, R_{DEF}.

Again, use the equation for parallel system.

2. Calculate the system reliability by considering blocks AB, C, DEF, G, and H, R_{tot}.

Use the series system equation.

Comment

The reliability of a component will frequently depend on the length of time it has been in service. Let T, the time to failure, be a random variable having its pdf specified by f(t). Then the probability that failure occurs in the time interval (0, t) is given by:

$$F(t) = \int_0^t f(t)\, dt$$

Let the reliability of the component be denoted by R(t). R(t) is the probability that the component survives to time t. Therefore,

$$R(t) = 1 - F(t)$$

The last equation establishes the relationship between the reliability of a component and its time to failure.

HAZARD AND RISK ASSESSMENT 5

Binomial Distribution

Introduction

Several probability distributions figure prominently in reliability calculations. The Binomial distribution is one of them. Consider n independent performances of a random experiment with mutually exclusive outcomes which can be classified "success" or "failure". The words "success" and "failure" are to be regarded as labels for two mutually exclusive categories of outcomes of the random experiment. They do not necessarily have the ordinary connotation of success or failure. Assume that P, the probability of success on any performance of the random experiment, is constant. Let q = 1 - P be the probability of failure. The probability distribution of X, the number of successes in n performances of the random experiment is the Binomial distribution with probability distribution function (pdf) specified by:

$$f(x) = P^x\ q^{n-x}\ n!/[x!(n - x)!] \ ;\ x = 0, 1, 2, \ldots , n$$

where: f(x) = probability of x successes in n performances
n = number of independent performances of a random experiment.

The Binomial distribution can therefore be used to calculate the reliability of a redundant system. A redundant system consisting of n identical components is a system which fails only if more than r components fail. Typical examples include single-usage equipment such as missile engines, short-life batteries and flash bulbs which are required to operate for one time period and not reused. Associate "success" with the failure of a component. Assume that the n components are independent with respect to failure, and that the reliability of each is 1 - P. Then X, the number of failures, has a Binomial pdf and the reliability of the random system is:

$$P(X <= r) = \sum_{x=0}^{r} P^x\ q^{n-x}\ n!/[x!(n - x)!]$$

Problem Statement and Data

A coolant sprinkler system in a reactor has 20 independent spray components each of which fails with probability of 0.1. The coolant system is considered to " fail" only if 4 or more of the sprays fail. What is the probability that the sprinkler system fails?

Solution

1. Let X denote the number of components which fail. Identify the value of n, P, and q from the problem statement.

Remember that:
n = number of components in the system
P = probability that a component fails
q = 1 - P.

2. Calculate the probability that the sprinkler system fails (i.e., P(X >= 4) by using the Binomial distribution equation.

For this calculation,

$$P(X >= 4) = \sum_{x=4}^{n} \{P^x q^{n-x} n!/ [x!(n - x)!]\}.$$

Note that calculation can be simplified by the fact that P(X >= 4) = 1 - P(X <= 3).

Comment

The reader is left the exercise of proving that for a Binomial distribution, the expected value (or mean) of the random variable X is nP and its variance is nPq.

HAZARD AND RISK ASSESSMENT 6

Weibull Distribution

Introduction

Frequently, the failure rate of equipment exhibits three stages: a break-in stage with a declining failure rate, a useful life stage characterized by a fairly constant failure rate, and a wearout period characterized by an increasing failure rate. A failure rate curve exhibiting these three phases is called a "bathtub curve". The Weibull distribution provides a mathematical model of all three stages of the bathtub curve. The probability distribution function (pdf) is given by:

$$f(t) = \alpha\beta t^{\beta-1} \exp\left(-\int_0^t \alpha\beta t^{-1}\right) dt$$

$$= \alpha\beta t^{\beta-1} \exp(-\alpha t^{\beta}) \quad ; \; t > 0, \alpha > 0, \beta > 0$$

where: α, β = constants
t = time.

Problem Statement and Data

Assume the time to failure (in hours), T, of a bus section in an electrostatic precipitator (ESP) has a Weibull distribution with $\alpha = 1.3 \times 10^{-3}$ and $\beta = 0.77$.

a. Find the probability that a bus section will fail in 1000 hours.
b. Develop an equation describing the probability of x bus section failures in 1000 hours in an ESP consisting of 100 bus sections.
c. Find the probability that a bus section fails between 70 and 1000 hours given that it has not failed in up to 70 hours.

Solution

1. First obtain the pdf for the bus section, f(t) by substituting the values of α and β given.

Use the Weibull distribution equation given above in the introductory section.

2. Calculate the probability that a bus section will fail in 1000 hours, i.e., P(T < 1000).

Note that:

$$P(T < 1000) = \int_0^{1000} f(t)\, dt$$

3. Develop the equation for the probability of x failures in 1000 hours for 100 bus sections by using the Binomial distribution.

Refer to Hazard and Risk Assessment Problem 5 for the details on the Binomial distribution.

a. First, determine the values of n, P, and q.

Note that P was calculated in step 2.

b. Obtain the equation for the probability of x failures in 1000 hours for 100 bus sections, f(x).

Note that
$f(x) = P^x q^{n-x} n!/[x!(n - x)!]$
; $x = 0, 1, ... , 100$

4. Calculate the probability that a bus section fails between 70 and 1000 hours given it has not failed in up to 70 hours (i.e., $P(70 < T < 1000 | T > 70)$.

Note that $P(B|A) = P(AB)/P(A)$. Also refer to Hazard and Risk Assessment problem 1.

Comment

The variety of assumptions about failure rate and the probability distribution of time to failure that can be accommodated by the Weibull distribution makes it specially attractive for the formulation of such assumptions.

Normal Distribution

Introduction

When time to failure, T, has a normal distribution, its probability distribution function (pdf) is given by:

$$f(t) = (1/\sqrt{2\pi}\sigma)\ \exp[-0.5((t-\mu)/\sigma)^2]\ ; -\infty < t < \infty$$

where: μ = mean value of T
σ = standard deviation of T.

Thus, if T is normally distributed with mean μ and standard deviation σ then the random variable, $(T-\mu)/\sigma$, is also normally distributed with mean 0 and standard deviation 1. The term $(T-\mu)/\sigma$ is called a "standard normal variable" (designated by Z) and the graph of its pdf is called a "standard normal curve." Appendix III provides a tabulation of areas under a standard normal curve to the right of z_0 for non-negative values of z_0. From this table, probabilities about a standard normal variable, Z, can be determined.

Problem Statement and Data

The measurement of the pitch diameter of the thread of a fitting is normally distributed with mean of 0.4008 inch and standard deviation of 0.0004 inch. The specifications are given as 0.4000 +/- 0.0010. What is the probability of a defective occurring?

Solution

1. In order to calculate the probability of a defective occurring, the probability of meeting the specification needs to be first calculated.

Let X denote the pitch diameter.

a. First determine the standard normal variable, Z.

$Z = (X-\mu)/\sigma$.

b. Determine the lower and upper limits of the probability of meeting specification.

The lower and upper limits are given by the specification.

c. Determine the probability of meeting specification, P_s, from the area under the standard normal curve between the lower and upper limits.

Note that $P_s = P((ll - \sigma)/\mu < Z < (ul - \sigma)/\mu)$
where: ll = lower limit
ul = upper limit.
Also refer to Appendix III.

2. Calculate the probability of a defective occurring, P_d.

Remember that $P_d = 1 - P_s$

Comment

The normal distribution can also be used to obtain probabilities concerning the mean, $\overline{X}$, of a sample of n observations on a random variable X. If X is normally distributed with mean μ and standard deviation σ, then $\overline{X}$, the sample mean, is normally distributed with mean μ and standard deviation $\sigma/\sqrt{n}$. If X is not normally distributed, then $\overline{X}$, the mean of a sample of n observations on X, is approximately normally distributed with mean μ and standard deviation $\sigma/\sqrt{n}$ provided the sample size n is large (greater than 30). This result is based on an important theorem in probability called the Central Limit Theorem. Also note that a non-negative random variable X has a log-normal distribution whenever ln X, the natural logarithm of X, has a normal distribution.

HAZARD AND RISK ASSESSMENT 8

Monte Carlo Simulation

Introduction

Monte Carlo simulation is a procedure for mimicking observations on a random variable that permits verification of results that would ordinarily require difficult mathematical calculations or extensive experimentation. The method uses computer programs called random number generators. A random number is a number selected from the interval (0,1) in such a way that the probabilities that the number comes from any two subintervals of equal length are equal. For example, the probability the number is in the subinterval (0.1,0.3) is the same as the probability that the number is in the subinterval (0.5,0.7). Thus, random numbers are observations on a random variable X having a uniform distribution on the interval (0,1). This means that the pdf of X is specified by:

$$f(x) = 1;\ 0 < x < 1$$
$$= 0;\ \text{elsewhere}$$

The above pdf assigns equal probability to subintervals of equal length in the interval (0,1). Using random number generators, Monte Carlo simulation can generate observed values of a random variable having any specified pdf. For example, to generate observed values of T, the time to failure, when T is assumed to have a pdf specified by f(t), first use the random number generator to generate a value of X between 0 and 1. The solution is an observed value of the random variable T having pdf specified by f(t).

Problem Statement and Data

One mechanical and two electrical components (A, B, C) in a spacecraft are connected in series. Assume that the individual component lifetimes are normally distributed with means and standard deviation given below.

	A	B	C
Mean (years)	100	90	80
Standard deviation (years)	30	20	10

Using the following random numbers, simulate the lifetime of the system and estimate its mean and standard deviation.

For A		For B		For C	
0.52	0.01	0.77	0.67	0.14	0.90
0.80	0.50	0.54	0.31	0.39	0.28
0.45	0.29	0.96	0.34	0.06	0.51
0.68	0.34	0.02	0.00	0.86	0.56
0.59	0.46	0.73	0.48	0.87	0.82

Solution

1. Let T_A, T_B, and T_C denote the lifetimes of components A, B, and C, respectively. Let T_S denote the lifetimes of the system.

Note that the components are connected in series.

a. First determine the values of the standard normal variable Z and T_A for component A using the 10 random numbers given.

Random No.	Z	T_A
0.52		
0.80		
0.45		
0.68		
0.59		
0.01		
0.50		
0.29		
0.34		
0.46		

The value of Z can be determined by realizing that the area under the standard normal distribution curve (or cumulative probability) is the random number generated. Use the standard normal cumulative probability table given in Appendix III. Also refer to Hazard and Risk Assessment problem 7.

Note that the standard normal distribution curve is symmetrical and the negative values of Z and the corresponding area are found by symmetry. For example,

$P(Z < -1.54) = 0.062$,
$P(Z > 1.54) = 0.062$, and
$P(0<Z<1.54) = 0.5 - P(Z>1.54)$
$= 0.5 - 0.062$
$= 0.438$

Also note that $Z = (T - \mu)/\sigma$ or alternatively, $T = \mu + \sigma Z$.

b. Next, determine the values of the standard normal variable Z and T_B for component B using the 10 random numbers given.

Random No.	Z	T_B
0.77		
0.54		
0.96		
0.02		
0.73		
0.67		
0.31		
0.34		
0.00		
0.48		

Same comment as step 1a applies.

c. Determine the values of the standard normal variable Z and T_C for component C using the 10 random numbers given.

Same comment as step 1a applies.

Random No.	Z	T_C
0.14		
0.39		
0.06		
0.86		
0.87		
0.90		
0.28		
0.51		
0.56		
0.82		

2. For each random value (10) of each component (A, B, C), determine the system lifetime, T_S.

Since the system is a series system, T_S is the minimum of the three lifetimes.

T_A	T_B	T_C	T_S

3. Calculate the mean value of T_S.

Refer to Hazard and Risk Assessment problem 3.

4. Calculate the standard deviation of T_S.

Refer to Hazard and Risk Assessment problem 3.

Comment

Monte Carlo simulation is an extremely powerful tool available to the scientist/engineer that can be used to solve multi-variable systems, ordinary and partial differential equations, numerical integrations, etc.

HAZARD AND RISK ASSESSMENT 9

Event Tree Analysis

Introduction

An event tree provides a diagrammatic representation of event sequences that begin with a so-called initiating event and terminate in one or more undesirable consequences. In contrast to a fault tree (considered in the next problem) which works backward from an undesirable consequence to possible causes, an event tree works forward from the initiating event to possible undesirable consequences. The initiating event may be equipment failure, human error, power failure or some other event that has the potential for adversely affecting an ongoing process.

Problem Statement and Data

If a building fire occurs, a smoke alarm sounds with probability 0.9. The sprinkler system functions with probability 0.7 whether or not the smoke alarm sounds. The consequences are minor fire damage (alarm sounds, sprinkler works), moderate fire damage with few injuries (alarm sounds, sprinkler falis), moderate fire damage with many injuries (alarm fails, sprinkler works), and major fire damage with many injuries (alarm fails, sprinker fails). Construct an event tree and indicate the probabilities for each of the four consequences.

Solution

1. Determine the first consequence(s) of the building fire and list the probabilities of the first consequence(s).	Begin by preparing a block diagram. Indicate the initiating event and each consequence as a block with a specified probability.
2. Determine the second consequence(s) of the building fire and the list the probabilities of the consequence(s).	This approach is typically used for event tree analysis and the same approach is recommended for this problem.
3. Determine the final consequence(s) and calculate the probabilities of the minor fire damage, moderate fire	Note that conditional probability applies. Also refer to Hazard and Risk Assessment Problem 1.

damage with few injuries,
moderate fire damage with many
injuries, and major fire
damage with many injuries.

Comment

Note that for each branch in an event tree, the sum of probabilities must equal 1.0. Note again that an event tree includes the following: (1) works forward from the initial event, or an event that has the potential for adversely affecting an ongoing process, and ends at one or more undesirable consequences, (2) is used to represent the possible steps leading to a failure or accident, (3) uses a series of branches which relate the proper operation and/or failure of a system with the ultimate consequences, (4) is a quick identification of the various hazards which could result from a single initial event, (5) is beneficial in examining the possibilities and consequences of a failure, (6) usually does not quantify (although it can) the potential of the event occurring, and (7) can be incomplete if all the initial events are not identified.

Thus, the use of event tree is sometimes limiting for hazard analysis because it lacks the capability of quantifying the potential of the event occurring. It may also be incomplete if all initial occurrences are not identified. Its use is beneficial in examining, rather than evaluating, the possibilities and consequences of a failure. For this reason, a fault tree analysis should supplement this model to establish the probabilities of the event tree branches. This topic is introduced in the next problem (Problem 10).

HAZARD AND RISK ASSESSMENT 10

Fault Tree Analysis

Introduction

Fault tree analysis seeks to relate the occurrence of an undesired event to one or more antecedent events. The undesired event is called the "top event" and the antecedent events are called "basic events." The top event may be, and usually is, related to the basic events via certain intermediate events. The fault tree diagram exhibits the casual chain linking the basic events to the intermediate events and the latter to the top event. In this chain the logical connection between events is illustrated by so called "logic gates". The principal logic gates are the **AND** gates symbolized on the fault tree by ⌂ and the OR gate symbolized by ⌂ .

Problem Statement and Data

A runaway chemical reaction can occur if coolers fail (A) or there is a bad chemical batch (B). Coolers fail only if both cooler #1 fails (C) and cooler #2 fails (D). A bad chemical batch occurs if there is a wrong mix (E) or there is a process upset (F). A wrong mix occurs only if there is an operator error (G) and instrument failure (H). Construct a fault tree. If the following annual probabilities are provided by the plant engineer, calculate the probability of a runaway chemical reaction occurring in a year's time.

P(C) = 0.05
P(D) = 0.08
P(F) = 0.06
P(G) = 0.03
P(H) = 0.01

Solution

1. Construct the fault tree, beginning from the top event.

The top event is the undesired event.

2. Obtain the first branch of the fault tree, applying the logic gates.

Remember that the runaway reaction occurs if coolers fail OR there is a bad chemical batch.

3. Obtain the second branch of the fault tree, applying the logic gates.

Remember that the coolers fail only if cooler #1 fails AND cooler #2 fails. A bad chemical batch occurs if there is a wrong mix OR there is a process upset.

4. Obtain the third branch of the fault tree, applying the logic gates.

Remember that a wrong mix occurs only if there is an operator error AND instrument failure.

5. Calculate the probability that the runaway reaction will occur.

When the logic gate of AND applies to a branch, the probability of the branch is the product of the block probabilities (refer to Hazard and Risk Assessment Problem 1). When the logic gate of OR applies to a branch, the probability of the branch is the sum of the block probabilities.

Comment

The reader should note that a fault tree includes the following: (1) works backward from an undesirable event or ultimate consequence to the possible causes and failures, (2) relates the occurrence of an undesired event to one or more preceding events, (3) "chain links" basic events to intermediate events which in turn connect the top event, (4) is used in the calculation of the probability of the top event, (5) is based on the most likely or credible events which lead to a particular failure or accident, and (6) analysis includes human error as well as equipment failure.

Thus, fault tree analysis (FTA) begins with the ultimate consequence and works backwards to the possible causes and failures. It is based on the most likely or credible events which lead to an accident. FTA demonstrates the mitigating or reducing effects and can also include human error as well as equipment failure and causes. The task of constructing a fault tree is a tedious job and requires a probability background to handle common mode failures, dependent events, and time constraints.