SOLUTIONS: HAZARD AND RISK ASSESSMENT

Hazard and Risk Assessment 1

1. First, determine the probability that the first item is defective, P(A).

   Since 10 out of 100 are defective,
   P(A) = (10/100)

2. Determine the probability that the second item is defective if the first is replaced, P(B).

   Since the first item is replaced, the probability for the defective item is the same.
   P(B) = (10/100)

3. Determine the probability that the two items are defective if the first is replaced,i.e., P(AB).

   P(AB) = P(A) P(B)
   = (10/100)(10/100)
   = 1/100

4. Determine the probability that the second item is defective if the first item is not replaced,i.e., P(B|A).

   Since the remaining lot contains 99 items,
   P(B|A) = 9/99

5. Determine the probability that the both items are defective if the first item is not replaced,i.e., P'(AB).

   P'(AB) = P(A) P(B|A)
   = (10/100)(9/99)
   = 1/110

Hazard and Risk Assessment 2

1. List the probabilities of the event of a policy holder being low risk, medium risk, and high risk.

   For low risk; $P(A_1) = 0.7$
   For medium risk; $P(A_2) = 0.2$
   For high risk; $P(A_3) = 0.1$

2. List the probabilities of the low, medium, and high risk policy holders filing claims within a given year.

   Letting B be the event of policy holder filing a claim,
   $P(B|A_1) = 0.0025$
   $P(B|A_2) = 0.01$
   $P(B|A_3) = 0.02$

3. Apply Bayes' Theorem to obtain the probability of claims being filed by the low risk policy holder, $P(A_1|B)$.

$$
\begin{aligned}
P(A_1|B) &= P(A_1)\ P(B|A_1)/[P(A_1)\ P(B|A_1) + P(A_2)\ P(B|A_2) + \\
&\quad P(A_3)\ P(B|A_3)] \\
&= (0.7)(0.0025)/[(0.7)(0.0025) + (0.2)(0.01) + \\
&\quad (0.1)(0.02)] \\
&= 0.30
\end{aligned}
$$

1. Calculate the probability that X will have a value between 2 and 6.

$$P(2 < X < 6) = \int_2^6 f(x)\,dx$$

$$= \int_2^6 1.7 \exp(-1.7x)\,dx$$

Since $\int \exp(ax)\,dx = (1/a)\exp(ax)$,

$$P(2 < X < 6) = -\exp(-1.7x)\Big|_2^6$$

$$= \exp[(-1.7)(2)] - \exp[(-1.7)(6)]$$
$$= 0.0333$$

2. Calculate the variance of X, σ^2.

   a. In order to calculate the variance, the average value of X or the expected value of X, E(X) needs to be calculated first.

$$E(X) = \int_0^\infty x\, f(x)\,dx$$

$$= \int_0^\infty x\,[1.7 \exp(-1.7x)]\,dx$$

From a table of integrals,

$$\int x \exp(ax)\,dx = (ax - 1)\exp(ax)/a^2$$

Therefore,

$$E(X) = (-1.7x - 1)\exp(-1.7x)/1.7\Big|_0^\infty$$

$$= 0 + (1/1.7)$$
$$= 0.5882$$

   b. Calculate the value of $E(X^2)$.

$$E(X^2) = \int_0^\infty x^2\,[1.7 \exp(-1.7x)]\,dx$$

From a table of integrals,

$$\int x^2 \exp(ax)\,dx = [x^2 \exp(ax)/a] - 2/a \int x \exp(ax)\,dx$$
$$= [x^2 \exp(ax)/a] - 2/a\,[(ax - 1)\exp(ax)/a^2]$$

Therefore,

$$E(X^2) = -x^2 \exp(-1.7x) + 2/1.7[(-1.7x-1)\exp(-1.7x)/1.7^2]\Big|_0^\infty$$

$= 2/(1.7)(1.7)$
$= 0.6920$

c. Calculate the variance, σ^2.

By definition: $\sigma^2 = E(X^2) - (E(X))^2$
$= 0.6920 - (0.5882)^2$
$= 0.3460$

Hazard and Risk Assessment 4

1. Since the system is a combined system of a series and parallel system, the parallel blocks should be combined into a single block first.

   a. Combine blocks A and B, i.e., calculate the combined reliability of A and B, R_{AB}.

   $$R_{AB} = 1 - (1 - R_A)(1 - R_B) = 1 - (1 - 0.7)(1 - 0.7) = 0.91$$

   b. Calculate the combined reliability of D, E, and F, R_{DEF}.

   $$R_{DEF} = 1 - (1 - R_D)(1 - R_E)(1 - R_F) = 1 - (1 - 0.6)(1 - 0.6)(1 - 0.6) = 0.936$$

2. Calculate the system reliability, R_{tot}, by considering blocks AB, C, DEF, G, and H.

   $$R_{tot} = R_{AB}\, R_C\, R_{DEF}\, R_G\, R_H = (0.91)(0.9)(0.936)(0.9)(0.9) = 0.621$$

Hazard and Risk Assessment 5

1. Let X denote the number of components which fail. Identify the value of n, P, and q from the problem statement.

   n = 20
   P = 0.1
   q = 0.9

2. Calculate the probability that the sprinkler system fails (i.e.,P(X >= 4) by using the Binomial distribution equation.

$$P(X >= 4) = \sum_{x=4}^{20} \{P^x q^{n-x} n!/[x!(n - x!)]\}.$$

   Note that calculation can be simplified by the fact that P(X >= 4) = 1 - P(X <= 3).

   Therefore,

$$\begin{aligned} P(X >= 4) &= 1 - P(X <= 3) \\ &= 1 - \sum_{x=0}^{3} (0.1)^x (0.9)^{20-x} 20!/[x!(20 - x)!] \\ &= 0.13 \end{aligned}$$

Hazard and Risk Assessment 6

1. First obtain the pdf for the bus section, f(t), by substituting the values of α and β given.

   $$f(t) = (1.3 \times 10^{-3})(0.77)\ t^{0.77-1} \exp(-1.3 \times 10^{-3} t^{0.77});\ t > 0$$

2. Calculate the probability that a bus section will fail within 1000 hours, P(T < 1000).

   $$P(T < 1000) = \int_0^{1000} f(t)\,dt$$
   $$= -\exp(-1.3 \times 10^{-3} t^{0.77}) \Big|_0^{1000}$$
   $$= 1 - \exp[-(1.3 \times 10^{-3})(1000)^{0.77}]$$
   $$= 0.23$$

3. Develop the equation for the probability of x failures in 1000 hours for 100 bus sections by using the Binomial distribution.

   a. First, determine the values of n, P, and q.

      n = 100
      P = 0.23
      q = 0.77

   b. Obtain the equation for the probability of x failures in 1000 hours for 100 bus sections, f(x).

      $$f(x) = P^x q^{n-x}\, n!/[x!(n-x)!]\ ;\ x = 0, 1, \ldots, 100$$
      $$= (0.23)^x (0.77)^{100-x}\, 100!/[x!(100-x)!]$$

4. Calculate the probability that a bus section fails between 70 and 1000 hours given it has not failed in up to 70 hours (i.e., P(70 < T < 1000| T > 70).

   $$P(70 < T < 1000 | T > 70) = P(70 < T < 1000)/P(T > 70)$$
   $$= \int_{70}^{1000} f(t)\,dt \Big/ \int_{70}^{\infty} f(t)\,dt$$
   $$= (0.97 - 0.77)/0.97$$
   $$= 0.21$$

1. In order to calculate the probability of a defective occurring, the probability of meeting specification needs to be first calculated.

   a. First determine the standard normal variable, Z.

   $$Z = (X - \mu)/\sigma$$
   $$= (X - 0.4008)/0.0004$$

   b. Determine the lower and upper limits of the probability of meeting specification.

   Lower limit (ll) = 0.4000 - 0.0010 = 0.3990
   Upper limit (ul) = 0.4000 + 0.0010 = 0.4010

   c. Determine the probability of meeting specification, P_s, from the area under the standard normal curve between the lower and upper limits.

   $$P_s = P[(ll-\mu)/\sigma < Z < (ul-\mu)/\sigma]$$
   $$= P\{[(0.3990 - 0.4008)/(0.0004)] < Z < (0.4010 - 0.4008)/(0.0004)]\}$$
   $$= P(-4.5 < Z < 0.5)$$

   From Appendix III,
   $$P_s = 0.69 - 0.00$$
   $$= 0.69$$

2. Calculate the probability of a defective occurring, P_d.

   $$P_d = 1 - P_s$$
   $$= 1- 0.69$$
   $$= 0.31$$

1. Let T_A, T_B, and T_C denote the lifetimes of components A, B, and C, respectively. Let T_S denote the lifetime of the system.

   a. First determine the values of standard normal variable Z and T_A for component A using the 10 random numbers given. Note that the random number generated is the cumulative probability shown in Appendix III. Also note that $T = \mu + \sigma Z$.

Random No.		Z		T_A
0.52	From Appendix A;	0.05	$T_A = 100 + 30(0.05) =$	102
0.80		0.84		125
0.45		-0.13		96
0.68		0.47		114
0.59		0.23		107
0.01		-2.33		30
0.50		0.00		100
0.29		-0.55		84
0.34		-0.41		88
0.46		-0.10		97

   b. Next, determine the values of standard normal variable Z and T_B for component B using the 10 random numbers given.

Random No.		Z		T_B
0.77	From Appendix III;	0.74	$T_B = 90 + 20(0.74) =$	105
0.54		0.10		92
0.96		1.75		125
0.02		-2.05		49
0.73		0.61		102
0.67		0.44		99
0.31		-0.50		80
0.34		-0.41		82
0.00		-3.90		12
0.48		-0.05		89

   c. Determine the values of standard normal variable Z and T_C for component C using the 10 random numbers given.

Random No.		Z		T_C
0.14	From Appendix III;	-1.08	$T_C = 80 + 10(-1.08) =$	69
0.39		-0.28		77
0.06		-1.56		64
0.86		1.08		91
0.87		1.13		91
0.90		1.28		93
0.28		-0.58		74
0.51		0.03		80
0.56		0.15		81
0.82		0.92		89

2. For each random value (10) of each component (A, B, C), determine the system lifetime, T_S.

T_A	T_B	T_C	T_S	
102	105	105	102	(minimum of A, B, and C)
125	92	77	77	
96	125	64	64	
114	49	91	49	
107	102	91	91	
30	99	93	30	
100	80	74	74	
84	82	80	80	
88	12	81	12	
97	89	89	89	
		Total =	635	

3. Calculate the mean value of T_S.

Mean = 635/10
= 63.5

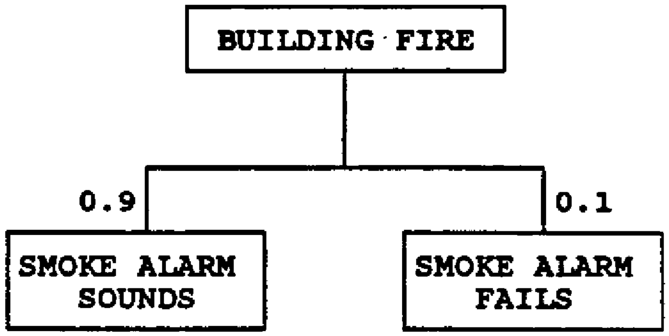
4. Calculate the standard deviation of T_S.

T_s		$(T_s - \mu)^2$
69	$(69 - 63.5)^2$ =	30.25
77		182.25
64		0.25
49		210.25
91		756.25
30		1122.25
74		110.25
80		272.25
12		2652.25
89		650.25
	Total =	5987.00

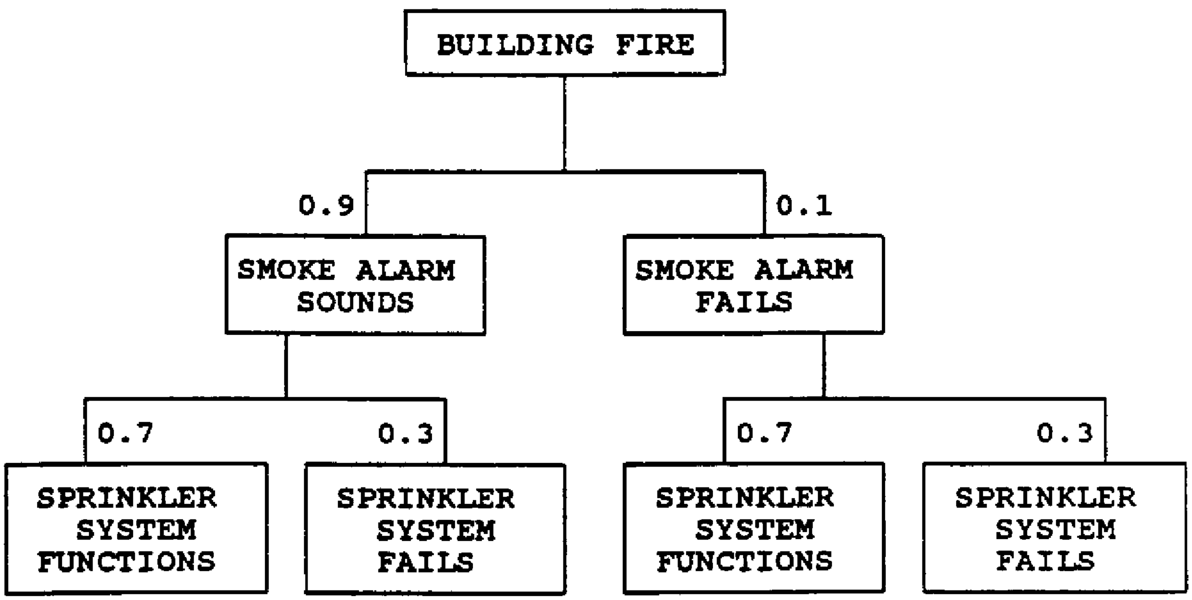
Standard deviation = $(5987/10)^{0.5}$
= 24.5

Hazard and Risk Assessment 9

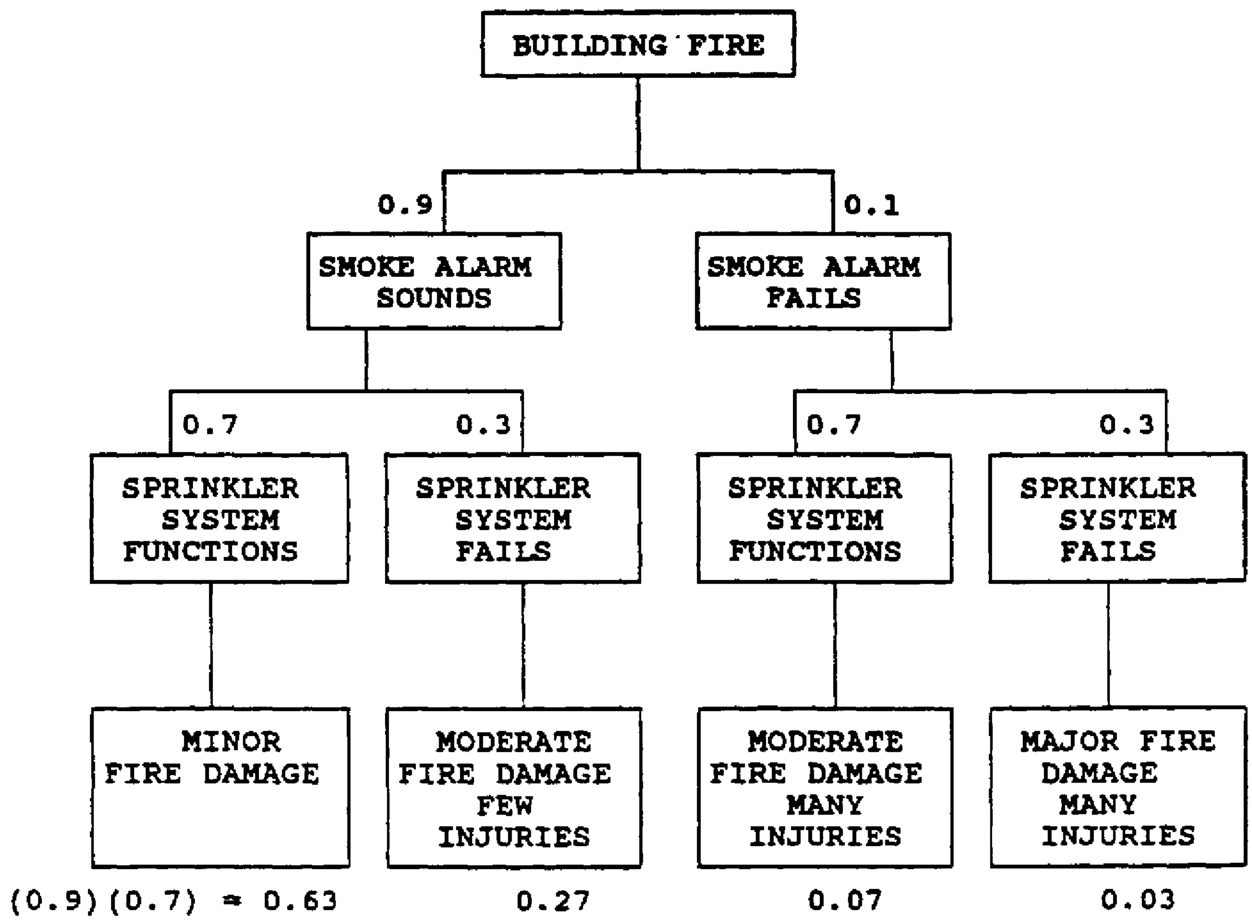
1. Determine the first consequence(s) of the building fire and list the probabilities of the first consequence.



2. Determine the second consequences of the building fire and the list the probabilities of the consequence(s).

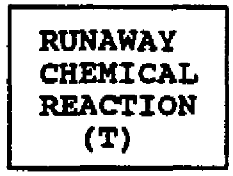


3. Determine the final consequences and calculate the probabilities of minor fire damage, moderate fire damage with few injuries, moderate fire damage with many injuries, and major fire damage with many injuries.

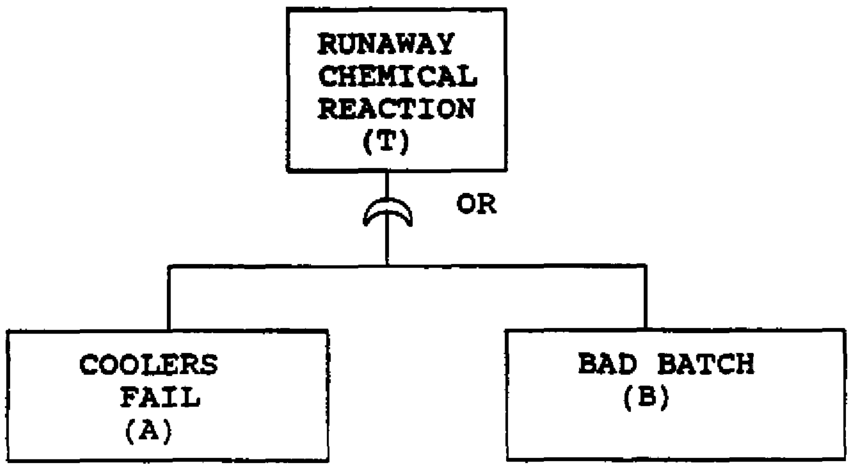


Hazard and Risk Assessment 10

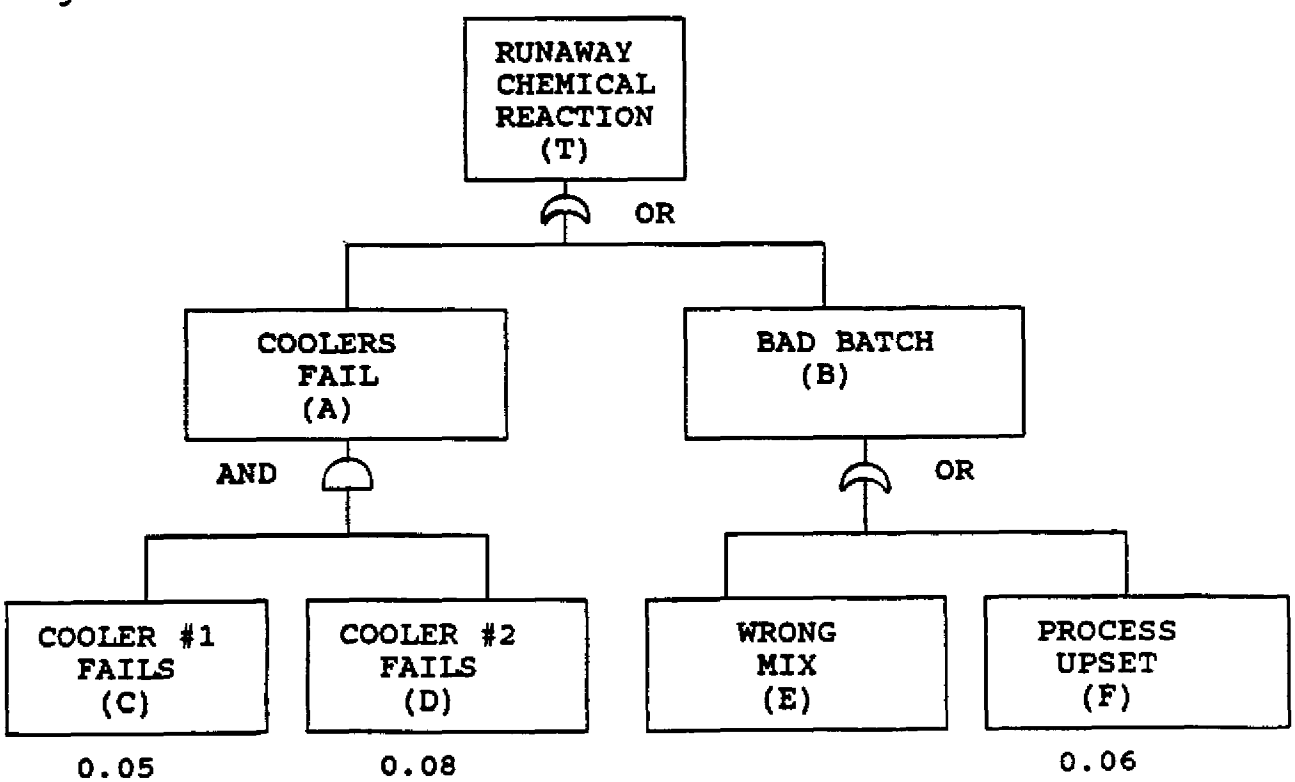
1. Construct the fault tree, beginning from the top event.



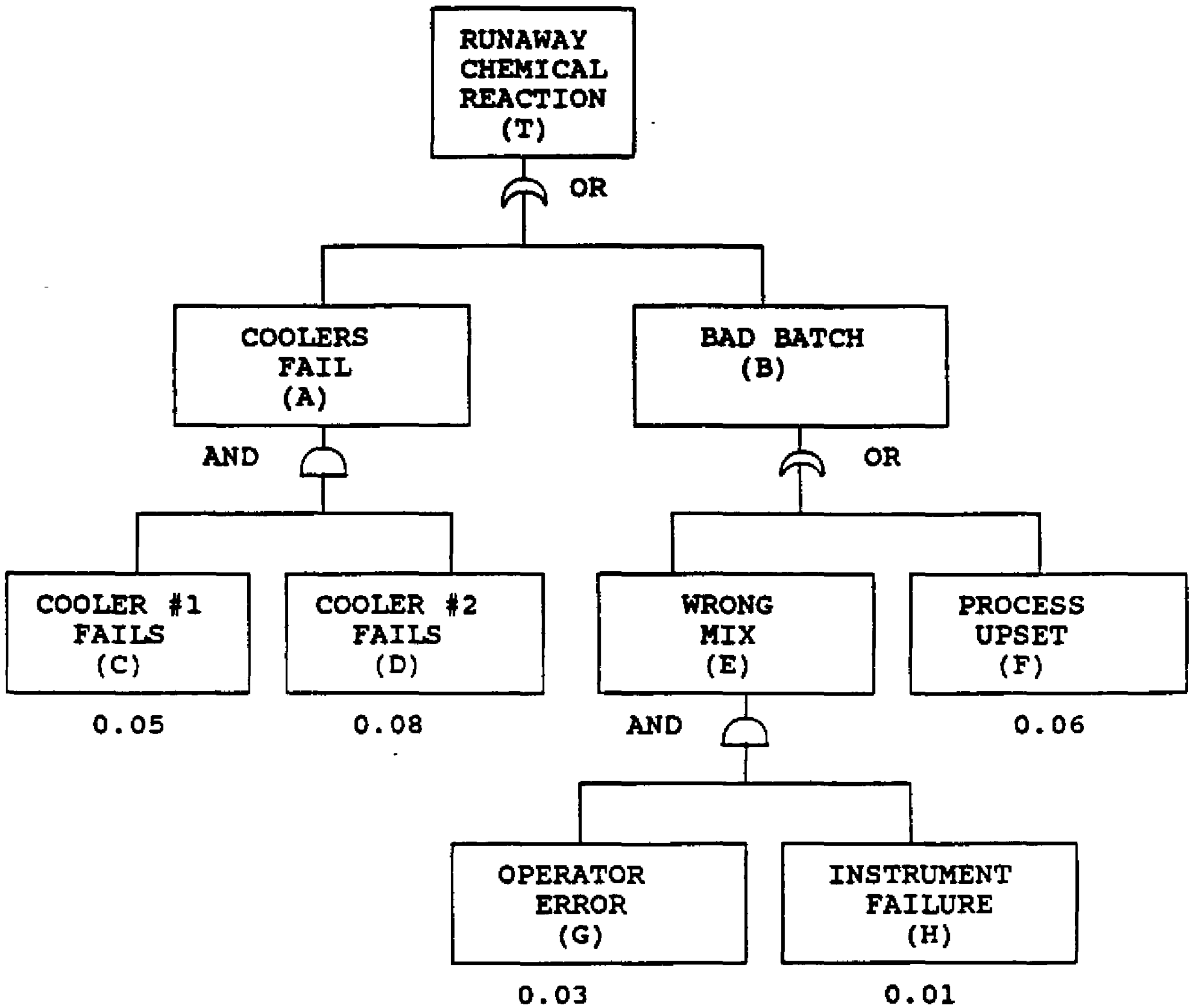
2. Obtain the first branch of the fault tree, applying the logic gates.



3. Obtain the second branch of the fault tree, applying the logic gates.



4. Obtain the third branch of the fault tree, applying the logic gates.



5. Calculate the probability that the runaway reaction will occur.

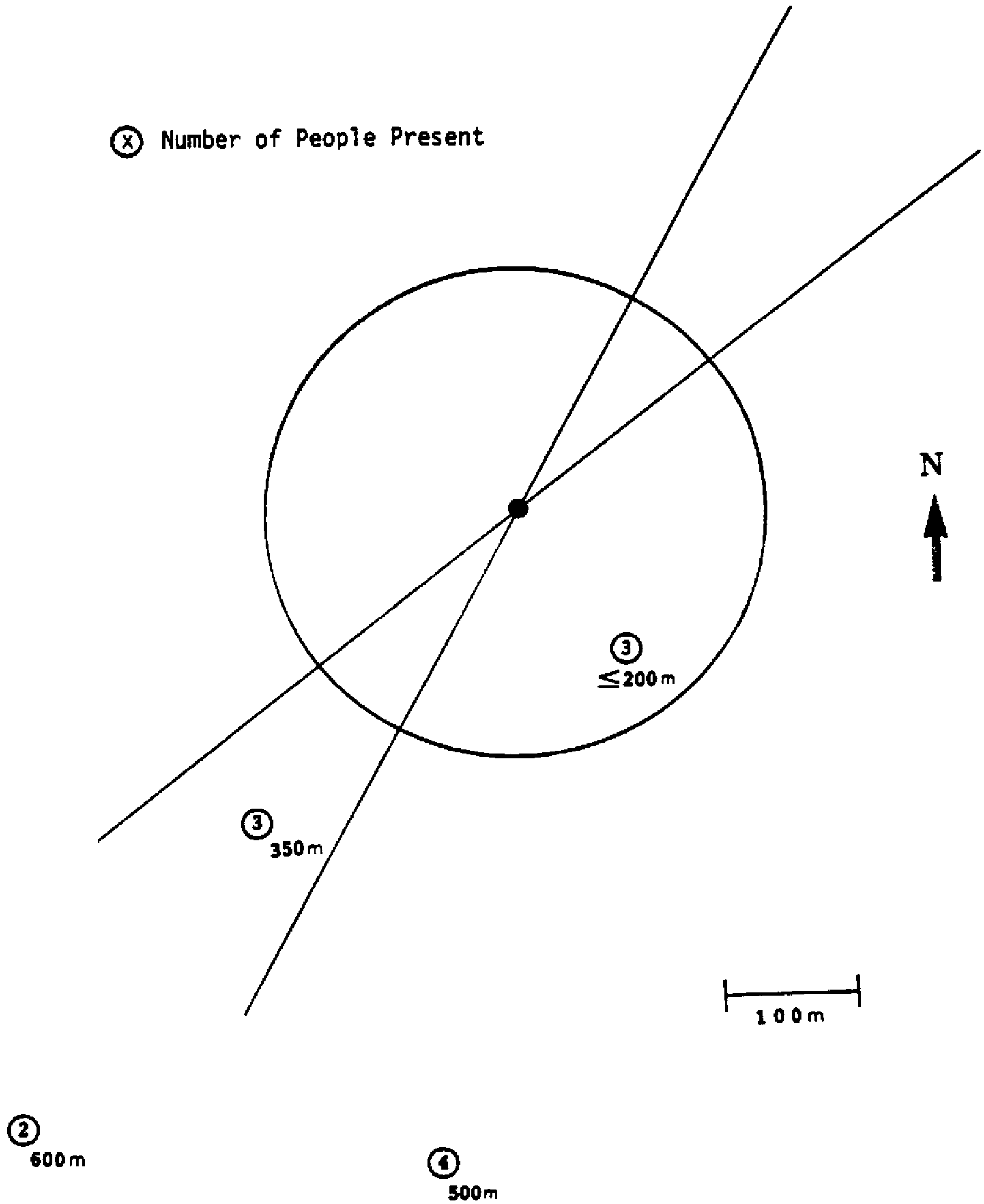
$$P = (0.05)(0.08) + (0.01)(0.03) + 0.06$$
$$= 0.0643$$

Note that the major contribution to the probability comes from F (process upset).

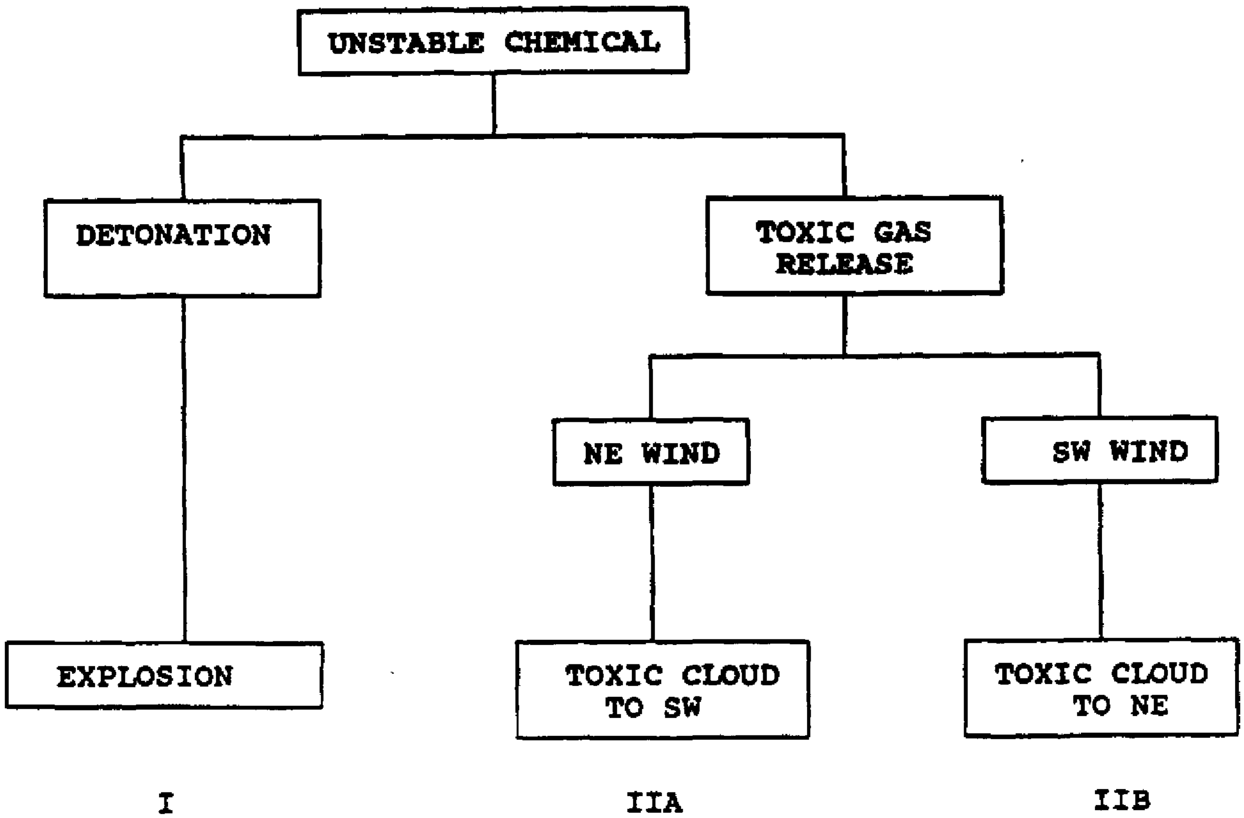
SOLUTIONS: DESIGN-ORIENTED APPLICATION PROBLEMS

Design-Oriented Application Problem 1

1. Draw a line diagram of the plant layout and insert all pertinent data and information.



2. Draw an event tree for the process.



3. Calculate the probability of event IIA occurring. Also calculate the probability of event IIB occurring.

$$P(IIA) = P(II)\ P(IIA|II) = (1/33{,}333)(0.33) = 1/100{,}000 = 10^{-5}$$

$$P(IIB) = P(II)\ P(IIB|II) = (1/33{,}333)(0.67) = 2/100{,}000 = 2 \times 10^{-5}$$

4. Perform a dispersion calculation to determine the zones where the concentration of ethylene oxide exceeds 0.33 ug/l. Assume a continuous emission for a point source.

   a. To maintain consistent units, convert wind speed from miles/hr to m/sec and concentration from ug/l to g/m^3.

$$\frac{3.3\ \text{ug}}{\text{l}} \cdot \frac{1\ \text{g}}{10^6\ \text{ug}} \cdot \frac{10^3\ \text{l}}{\text{m}^3} = 3.3 \times 10^{-4}\ \text{g/m}^3$$

$$\frac{60\ \text{miles}}{\text{hr}} \cdot \frac{5280\ \text{ft}}{\text{mile}} \cdot \frac{\text{hr}}{3600\ \text{sec}} \cdot \frac{0.3048\ \text{m}}{\text{ft}} = 2.68\ \text{m/sec}$$

   b. Set up the Pasquill-Gifford model using the data and calculations provided above.

$$C(x,0,0,H) = m \exp[-0.5(H/\sigma_z)^2]/\pi \sigma_y\ \sigma_z\ u$$

$C = 3.3 \times 10^{-4}$ g/m^3
$m = 240$ g/sec
$u = 2.68$ m/sec
$H = 125$ m
$\sigma_y,\ \sigma_z = f(x)$

   c. Calculate the downwind concentration that satisfies the Pasquill-Gifford equation. See "hint" in problem statement section for the possible values of the downwind distance x. Note that the concentration goes through a maximum that is in excess of 0.33 ug/l; thus, there are two solutions.

   The results for the recommended downwind distances are tabulated below.

x (m)	σ_y (m)	σ_z (m)	C (g/m^3)
300	52	30	3.20×10^{-6}
500	83	51	3.36×10^{-4}
800	128	86	9.10×10^{-4}
1000	156	110	8.72×10^{-4}
1500	225	170	5.72×10^{-4}
2000	295	235	3.59×10^{-4}

A linear interpolation (or plotting the results on a graph) indicates that the maximum GLC is approximately 9.9×10^{-4} g/m^3 and is located at a downwind distance of about 850 m. The "critical" zone is located between 500 m and 2175 m.

5. Determine which individuals within the pie shaped segment downwind from the source will be killed if either accident (I or II) occurs. Refer to problem statement or the line diagram obtained in step 1.

   Three (3) individuals within the 200 m radius will die from accident I.
   Two (2) individuals located in the pie-shaped segment and 600 m southwest of the emission source will die from Accident II.

6. Determine the total annual risk, TAR, for the process. The total risk, measured in terms of the average annual total number of people killed, is obtained by multiplying the number of people in each impact zone by the sum of the probabilities of the events affecting that zone, and summing the results.

$$
\begin{aligned}
TAR &= (3)P(I) + (2)P(IIA) \\
&= (3)(10^{-6}) + (2)(10^{-5}) \\
&= 2.3 \times 10^{-5}
\end{aligned}
$$

7. Calculate the average annual risk, AAR, based only on the "potentially affected" people. The average annual individual risk for the 8 people in the impact zone is obtained by dividing the total annual risk by 8.

   Since only eight people are "affected",

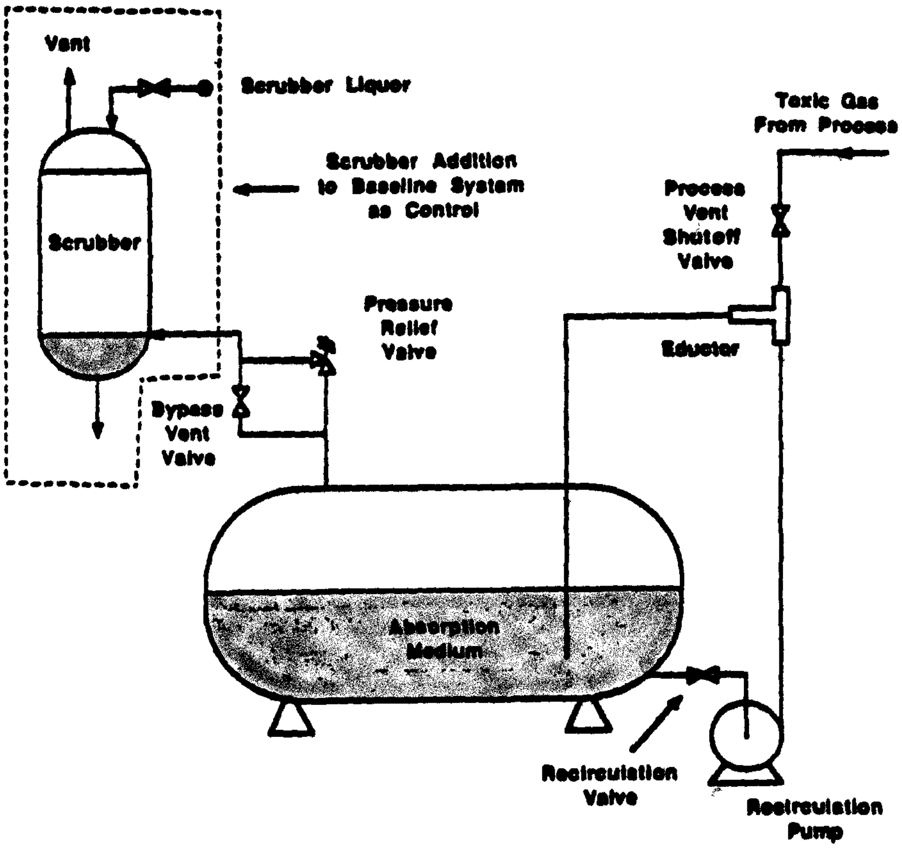
$$
\begin{aligned}
AAR &= 2.3 \times 10^{-5}/8 \\
&= 2.9 \times 10^{-6}
\end{aligned}
$$

8. Calculate the average annual individual risk for all the individuals within the plant boundary. The average is now based on 12 rather than 8 individuals.

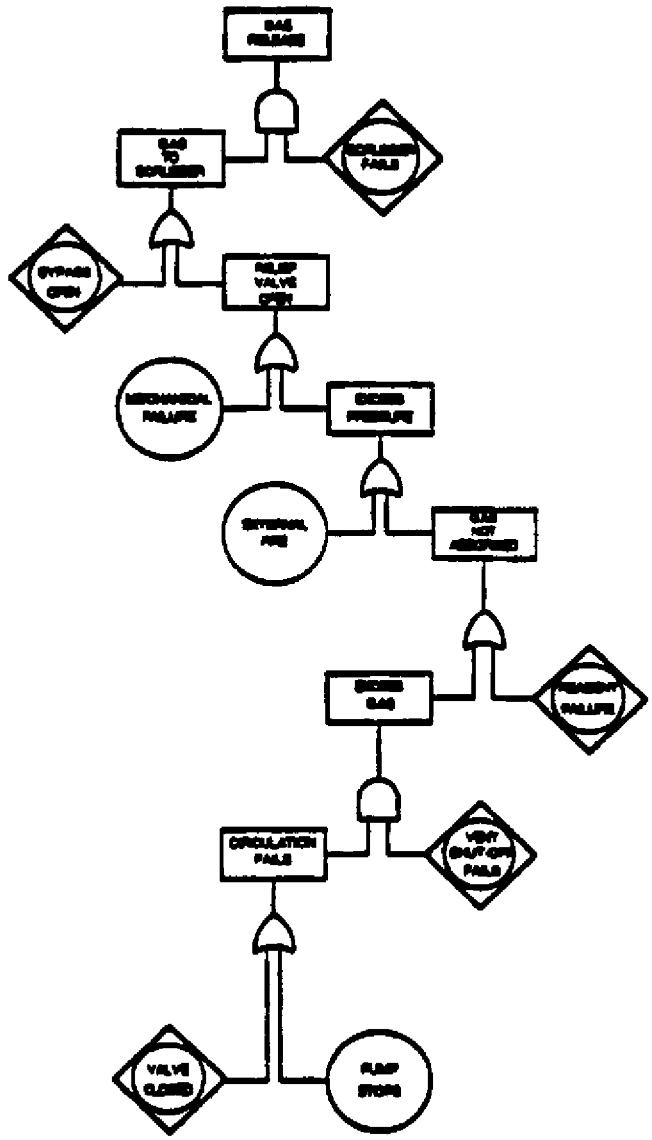
$$
\begin{aligned}
AAR' &= 2.3 \times 10^{-5}/12 \\
&= 1.9 \times 10^{-6}
\end{aligned}
$$

Design-oriented Application Problem 2

1. Develop a line diagram of the process.

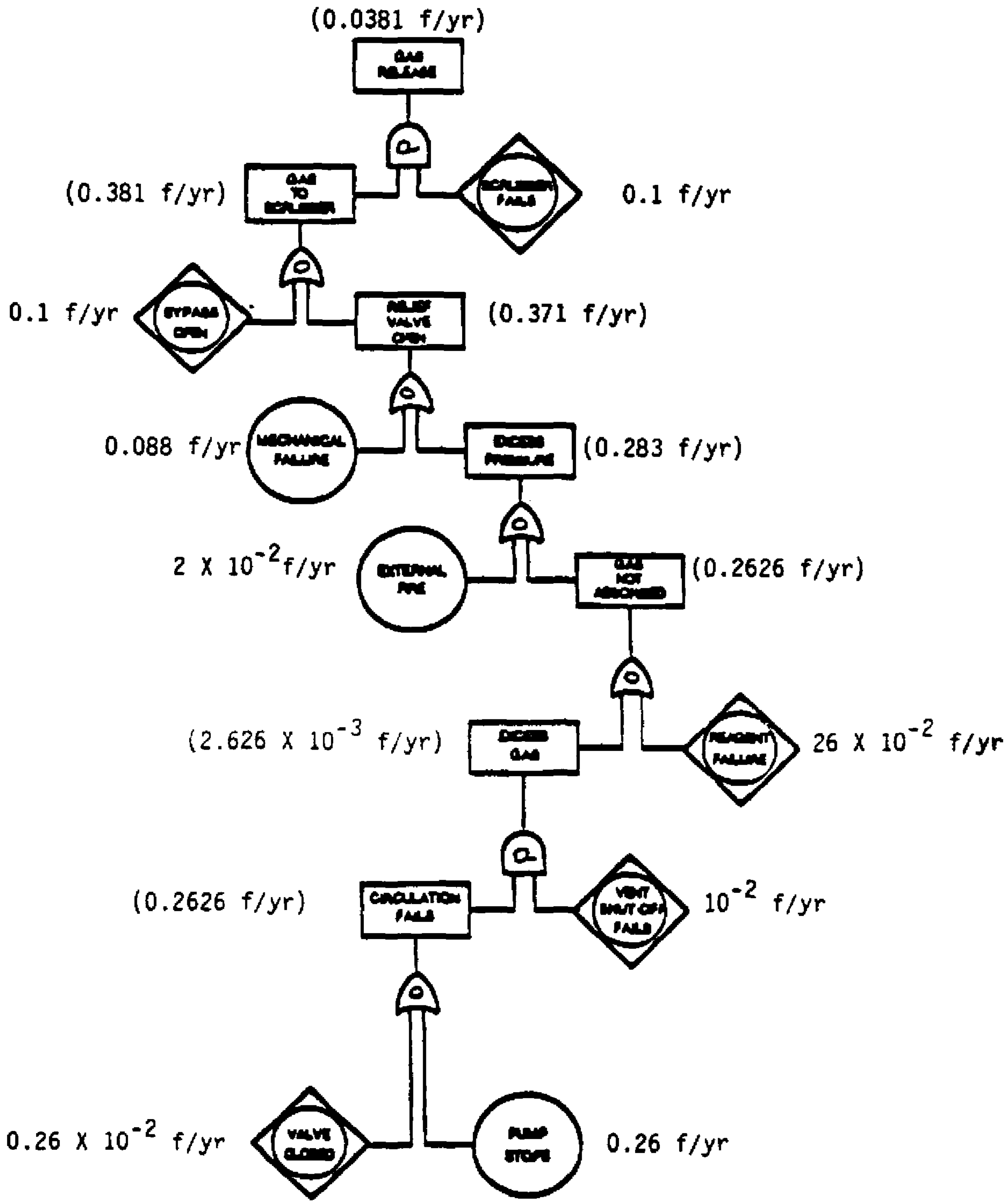


2. Develop a fault tree diagram for the above process, includir the scrubber.



3. Using the annual failure frequency data provided in the problem statement, in conjunction with the fault tree diagram, the failure probability calculations in Table 1 are performed and the results are inserted in the fault tree diagram (see Figure S-1).

   The fault tree calculations presented in Table S-1 are performed for the case of "scrubber only". The similar approach can be used for the baseline, the higher level, and the higher level/scrubber cases.



Note: Values in parenthesis
are calculated values.

Figure S-1. Fault Tree with Failure Probability

Table S-1. Annual Failure Probabilities

Failure Event	Baseline	Scrubber Only	Higher Level	Higher Level & Scrubber
Pump Stops	0.26	0.26	1.30×10^{-5}	1.30×10^{-5}
Circulation Valve Closed	2.60×10^{-3}	2.60×10^{-3}	1.30×10^{-3}	1.30×10^{-3}
Circulation Fails	2.63×10^{-1}	2.63×10^{-1}	1.31×10^{-3}	1.31×10^{-3}
Process Vent Shutoff Fails	1.00×10^{-2}	1.00×10^{-2}	5.24×10^{-6}	5.24×10^{-6}
Reagent Failure	0.26	0.26	1.30×10^{-3}	1.30×10^{-3}
Excess Gas	2.63×10^{-3}	2.63×10^{-3}	5.24×10^{-6}	5.24×10^{-6}
Gas Not Absorbed	2.63×10^{-1}	2.63×10^{-1}	1.30×10^{-3}	1.30×10^{-3}
External Fire	2.00×10^{-2}	2.00×10^{-2}	2.00×10^{-3}	2.00×10^{-3}
Excess Pressure	2.83×10^{-1}	2.83×10^{-1}	3.30×10^{-3}	3.30×10^{-3}
Relief Valve Failure	8.80×10^{-2}	8.80×10^{-2}	4.40×10^{-2}	4.40×10^{-2}
Release Thru Relief Valve (Open)	3.71×10^{-1}	3.71×10^{-1}	4.73×10^{-2}	4.73×10^{-2}
Release Thru Bypass (Open)	1.00×10^{-2}	1.00×10^{-2}	5.00×10^{-3}	5.00×10^{-3}
Gas To Scrubber	-	3.81×10^{-1}	5.23×10^{-2}	5.23×10^{-2}
Scrubber Failure	-	0.10	-	5.00×10^{-2}
Gas Release To Environment	3.81×10^{-1}	3.81×10^{-2}	5.23×10^{-2}	2.62×10^{-3}
Average Expected Time Between Releases, Years[a]	2.62	26.2	19.1	382

[a] Average interval between releases = 1/(release frequency)

4. Complete the cost control table. Note that the cost effectiveness is defined as the ratio of the additional cost in dollars on an annual basis from the baseline case to the difference between the average interval between releases and the baseline case.

For "scrubber only" case:

$$CRF = (i)(1.0 + i)^n/[(1.0 + i)^n - 1]$$
$$= (0.1)(1.0 + 0.1)^{10}/[(1.0 + 0.1)^{10} - 1]$$
$$= 0.1627$$

Annual capital cost = (capital cost)(CRF)
= (278,000)(0.1627)
= 46,480 $

Total annual cost = annual capital cost + O & M cost
= 46,480 + 26,800
= 73,280 $

Incremental total annual cost = total annual cost - baseline total annual cost
= 73,280 - [(0.1627)(236,000) + 24,460]
= 9,360 $

Cost effectiveness = (9,360)/(26.2 - 2.6)
= 397 $/release free yr

The similar calculation can be performed for the baseline, the scrubber only, higher level, and higher level/scrubber cases. The results are shown in Table S-2.

Table S-2. Summary of Total System Costs for Baseline, Scrubber Only, Higher Level Controls, and Both Scrubber and Higher Level Controls

	Baseline	Scrubber Only	Higher Level	Higher Level & Scrubber
Capital Cost ($)				
Equipment	114,000	134,000	122,300	142,300
Installation	49,000	58,000	52,000	61,000
Indirect	73,000	86,000	78,000	91,000
Total	236,000	278,000	252,000	294,000
O & M Cost ($/yr)				
Labor	6,000	6,000	6,000	6,000
Maintenance	8,000	9,500	8,500	10,000
Other Direct	960	1,130	1,100	1,280
Indirect	9,500	10,170	10,330	11,000
Total	24,460	26,800	25,940	28,280
Capital Recovery Factor (@ 10 %, 10 yrs)	0.1672	0.1672	0.1672	0.1672
Annual Capital Cost ($/yr)	39,460	46,480	42,180	49,210
Total Annual Cost ($/yr)	63,920	73,280	68,120	77,490
Incremental Total Annual Cost ($/yr)	-	9,360	4,200	13,570
Release Freq. (releases/yr)[a]	3.81×10^{-1}	3.81×10^{-2}	5.23×10^{-2}	2.62×10^{-3}
Average Interval Between Releases (yr)[a]	2.62	26.2	19.1	382
Cost Effectiveness ($/release free yr)	-	397	255	35.8

[a] See Table S-1.

5. Comment on the cost effectiveness of the three proposed control options.

   Based on the above definition of cost effectiveness. employing both the packed-bed scrubber and higher level controls will provide the company with the "best" solution. Using both levels of control will annually cost the company $35 for each incremental release-free year.